

非线性物理混沌现象

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- 实验原理
- 数值模拟与混沌频谱分析
- 混沌现象观测
- 非线性负阻I-V特性
- 元件参数测量和非线性方程求解
- C调制
- 混沌通讯

chua 电路方程

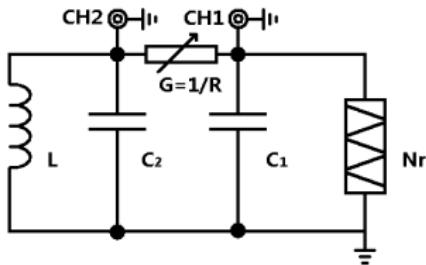


Fig: chua 电路示意图

chua 电路方程

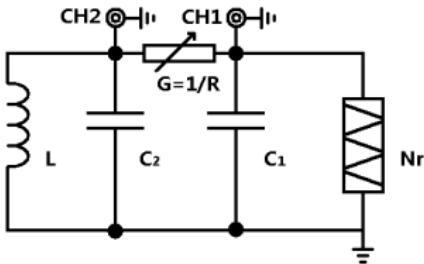


Fig: chua 电路示意图

$$\begin{cases} C_1 \frac{dU_1}{dt} = G(U_2 - U_1) - g(U_1) \\ C_2 \frac{dU_2}{dt} = G(U_1 - U_2) + I_L \\ L \frac{dI_L}{dt} = -U_2 \end{cases} \quad (1)$$

$$g(U) = G_b U + \frac{G_b - G_a}{2}(|U - E| - |U + E|) \quad (2)$$

chua 电路方程解

$$\begin{cases} \dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{b} \\ \mathbf{X}(0) = \mathbf{X}_0 \end{cases} \quad (3)$$

$$\lambda_i = \gamma, \sigma \pm i\omega \quad (4)$$

chua 电路方程解

$$\begin{cases} \dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{b} \\ \mathbf{X}(0) = \mathbf{X}_0 \end{cases} \quad (3)$$

$$\lambda_i = \gamma, \sigma \pm i\omega \quad (4)$$

$$\begin{cases} \mathbf{X}_Q = -\mathbf{A}^{-1}\mathbf{b} \\ \mathbf{x}_r(t) = c_r e^{\gamma t} \vec{\xi}_\gamma \\ \mathbf{x}_c(t) = 2c_c e^{\sigma t} [\cos(\omega t + \psi_c) \vec{\eta}_r - \sin(\omega t + \psi_c) \vec{\eta}_i] \end{cases} \quad (5)$$

$$\mathbf{X}(t) = \mathbf{X}_Q + \mathbf{x}_r(t) + \mathbf{x}_c(t) \quad (6)$$

chua 电路方程简化

$$\left\{ \begin{array}{l} x = \frac{U_1}{E} \quad y = \frac{U_2}{E} \quad z = \frac{I_L}{EG} \\ \alpha = \frac{C_2}{C_1} \quad \beta = \frac{C_2}{LG^2} \quad \tau = \frac{G}{C_2} t \\ a = \frac{G_a}{G} \quad b = \frac{G_b}{G} \end{array} \right. \quad (7)$$

chua 电路方程简化

$$\begin{cases} x = \frac{U_1}{E} & y = \frac{U_2}{E} & z = \frac{I_L}{EG} \\ \alpha = \frac{C_2}{C_1} & \beta = \frac{C_2}{LG^2} & \tau = \frac{G}{C_2} t \\ a = \frac{G_a}{G} & b = \frac{G_b}{G} \end{cases} \quad (7)$$

$$\begin{cases} C_1 \frac{dU_1}{dt} = G(U_2 - U_1) - g(U_1) \\ C_2 \frac{dU_2}{dt} = G(U_1 - U_2) + I_L \\ L \frac{dI_L}{dt} = -U_2 \end{cases} \Rightarrow \begin{cases} \frac{dx}{d\tau} = \alpha[y - x - f(x)] \\ \frac{dy}{d\tau} = x - y + z \\ \frac{dz}{d\tau} = -\beta y \end{cases} \quad (8)$$

chua 电路方程简化

$$\begin{cases} x = \frac{U_1}{E} & y = \frac{U_2}{E} & z = \frac{I_L}{EG} \\ \alpha = \frac{C_2}{C_1} & \beta = \frac{C_2}{LG^2} & \tau = \frac{G}{C_2} t \\ a = \frac{G_a}{G} & b = \frac{G_b}{G} \end{cases} \quad (7)$$

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$$f(x) = bx + \frac{a-b}{2}(|x+1| - |x-1|) = \begin{cases} bx + a - b & x \geq 1 \\ ax & -1 < x < 1 \\ bx - a + b & x \leq -1 \end{cases} \quad (9)$$

chua电路的不动点

$$\begin{pmatrix} \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{pmatrix} = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha \begin{pmatrix} f(x) \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

chua 电路的不动点

$$\begin{pmatrix} \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{pmatrix} = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha \begin{pmatrix} f(x) \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

$$\begin{cases} x + f(x) = 0 \\ y = 0 \\ z + z = 0 \end{cases} \Rightarrow \mathbf{x}_Q = \begin{cases} (k, 0, -k) & x \geq 1 \\ (0, 0, 0) & |x| < 1 \\ (-k, 0, k) & x \leq 1 \end{cases} \quad (11)$$

chua 电路的不动点

$$\begin{pmatrix} \frac{dx}{d\tau} \\ \frac{dy}{d\tau} \\ \frac{dz}{d\tau} \end{pmatrix} = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha \begin{pmatrix} f(x) \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

$$\begin{cases} x + f(x) = 0 \\ y = 0 \\ z + z = 0 \end{cases} \Rightarrow \mathbf{x}_Q = \begin{cases} (k, 0, -k) & x \geq 1 \\ (0, 0, 0) & |x| < 1 \\ (-k, 0, k) & x \leq 1 \end{cases} \quad (11)$$

$$k = \frac{b-a}{b+1} = \frac{G_b - G_a}{G_b + G} \quad (12)$$

变换矩阵的本征值

$$\begin{cases} \mathbf{X} = (x, y, z) \\ \mathbf{Q} = (k, 0, -k) \\ s = \begin{cases} a & -1 < x < 1 \\ b & |x| \geq 1 \end{cases} \end{cases} \quad (13)$$

变换矩阵的本征值

$$\begin{cases} \mathbf{X} = (x, y, z) \\ \mathbf{Q} = (k, 0, -k) \\ s = \begin{cases} a & -1 < x < 1 \\ b & |x| \geq 1 \end{cases} \end{cases} \quad (13)$$

$$\mathbf{M} = \begin{pmatrix} -\alpha(s+1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix}$$

变换矩阵的本征值

$$\begin{cases} \mathbf{X} = (x, y, z) \\ \mathbf{Q} = (k, 0, -k) \\ s = \begin{cases} a & -1 < x < 1 \\ b & |x| \geq 1 \end{cases} \end{cases} \quad (13)$$

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变换矩阵的本征值

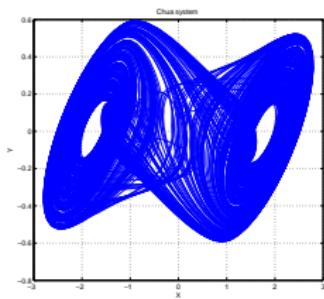
$$\begin{cases} \mathbf{X} = (x, y, z) \\ \mathbf{Q} = (k, 0, -k) \\ s = \begin{cases} a & -1 < x < 1 \\ b & |x| \geq 1 \end{cases} \end{cases} \quad (13)$$

$$\mathbf{M} = \begin{pmatrix} -\alpha(s+1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \Rightarrow \frac{d\mathbf{x}}{d\tau} = \begin{cases} \mathbf{M}(\mathbf{X} - \mathbf{Q}) & x \geq 1 \\ \mathbf{M}\mathbf{X} & |x| < 1 \\ \mathbf{M}(\mathbf{X} + \mathbf{Q}) & x \leq 1 \end{cases} \quad (14)$$

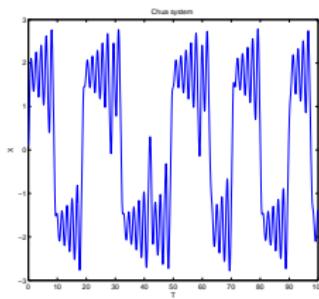
$$|\lambda \mathbf{I} - \mathbf{M}| = \lambda^3 + (1 + \alpha + s\alpha)\lambda^2 + (\beta + s\alpha)\lambda + \alpha\beta(1 + s) \quad (15)$$

- 典型混沌信号

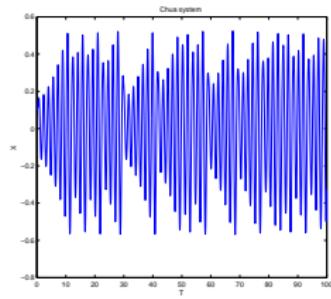
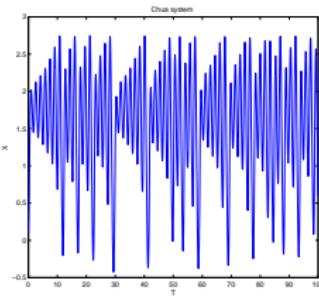
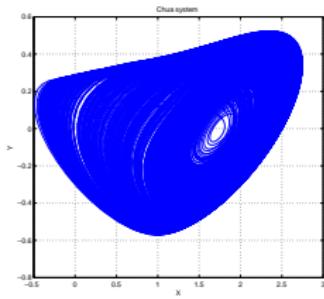
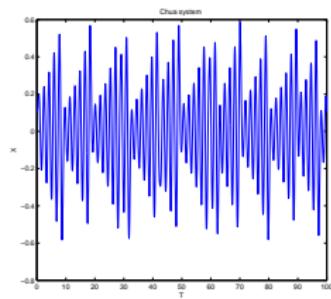
相图



U_1

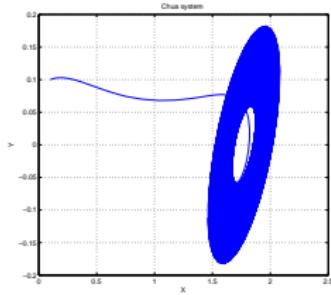
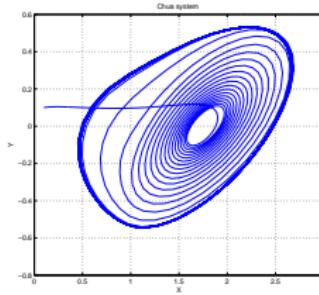
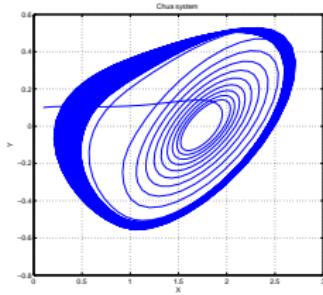
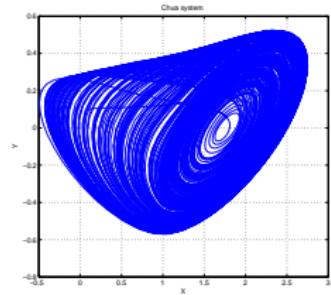
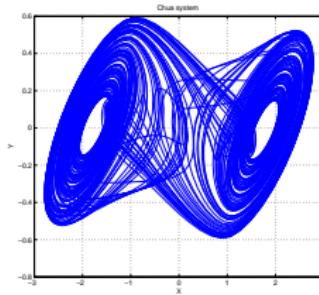
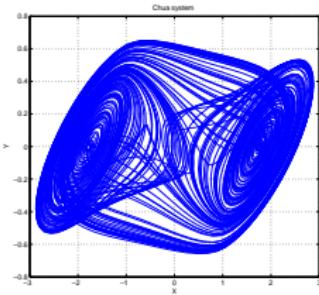


U_2



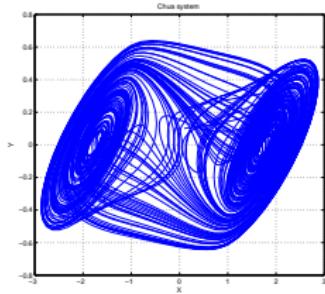
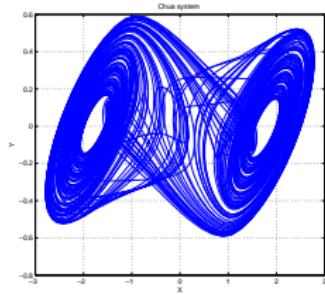
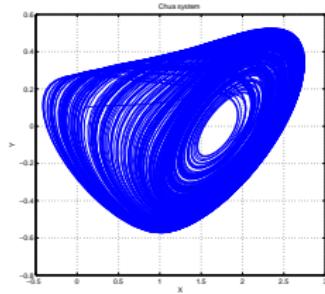
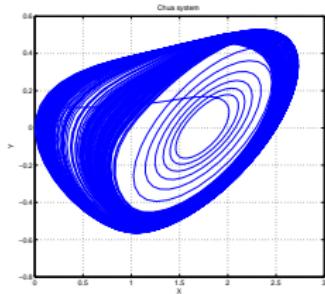
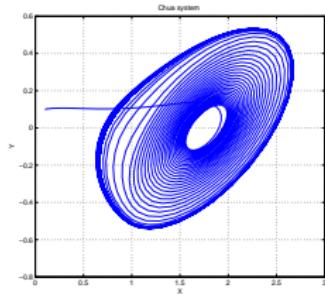
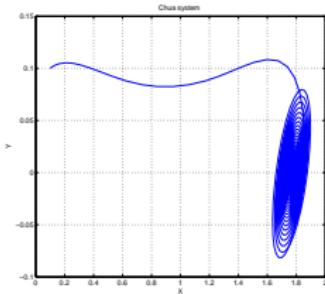
数值模拟

- G调制(从大到小)

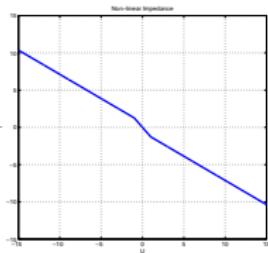


数值模拟

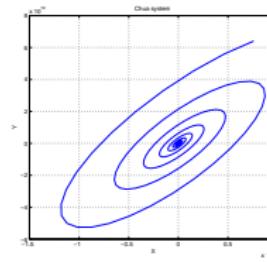
- C调制(C_1 从大往小)



- 数值模拟中遇到的发散问题



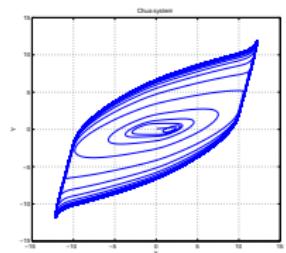
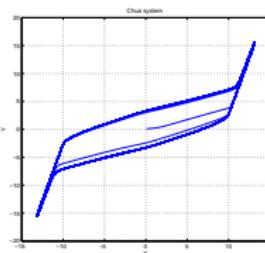
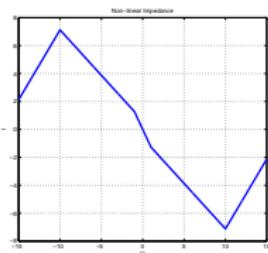
原始非线性负载IV特性



相图发散

数值模拟

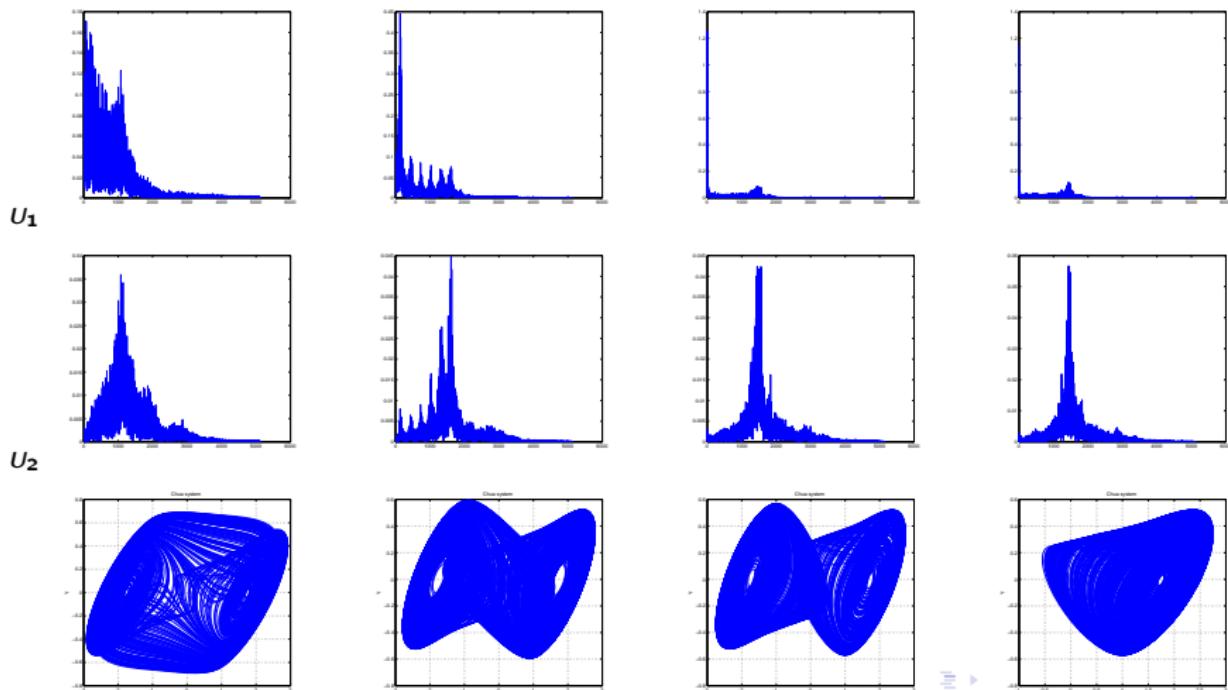
- 发散由于不适合的非线性负载模型所致



修改非线性负载IV特性 初始(远离双吸引子) 初始(靠近双吸引子)

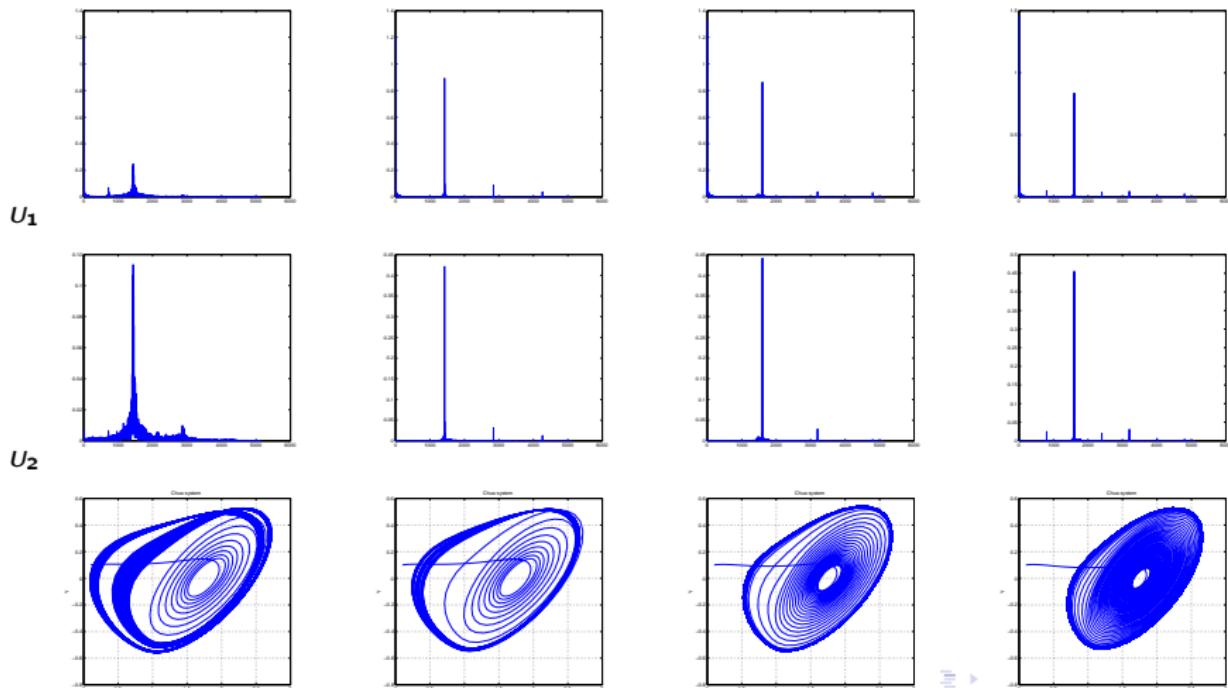
时域信号的频谱分析

- 初始状态至不动点, G调制(从大到小)



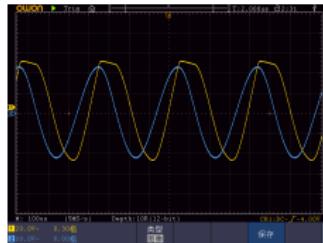
时域信号的频谱分析

- 采样点数 $N = 51.2k$ 采样频率 $F_s = 51.2k$

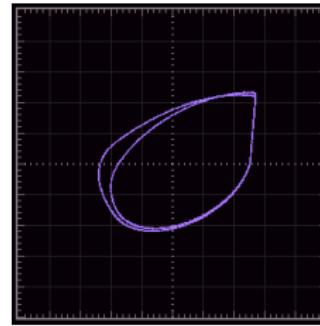
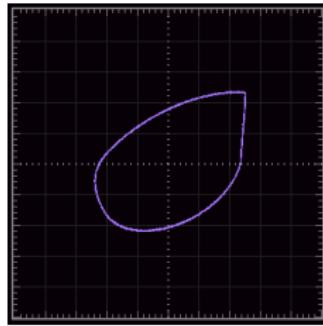
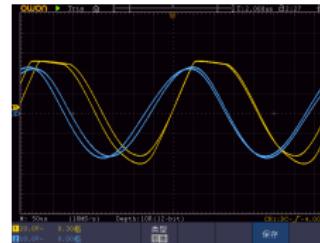


混沌现象

单周期

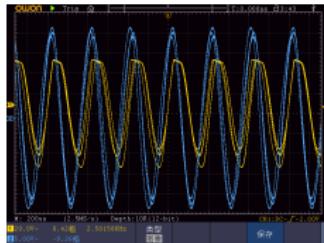


双周期

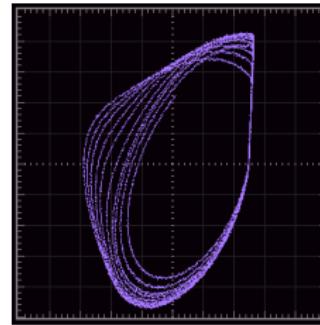
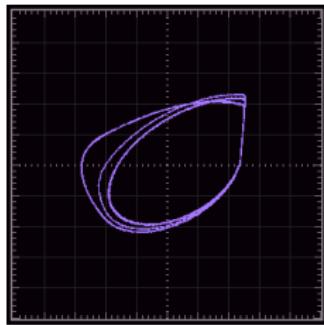
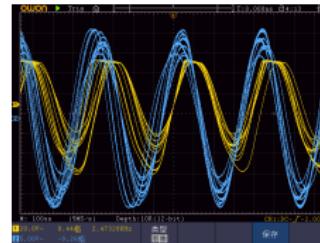


混沌现象

三周期

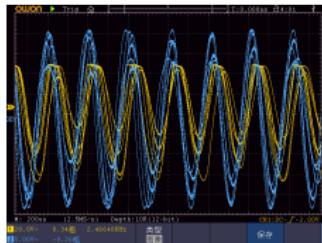


阵发混沌

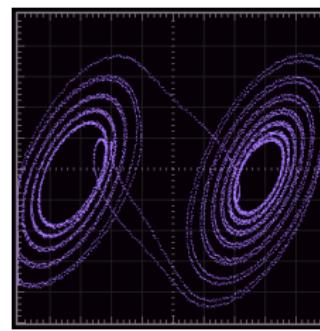
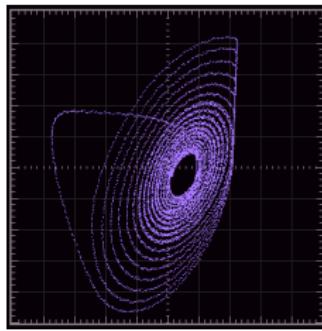
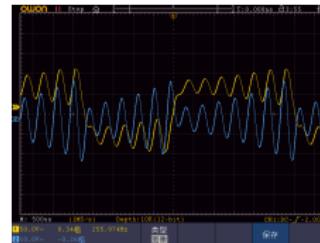


混沌现象

单吸引子

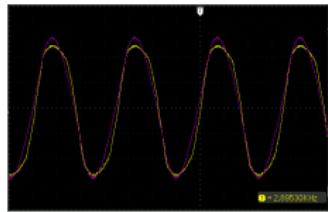


双吸引子

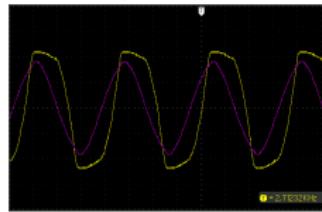


混沌现象

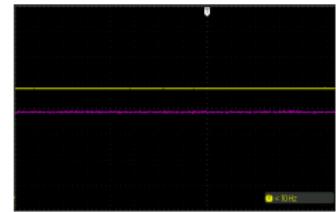
初始状态



初始状态(远离双吸引子)



不动点



非线性负阻的IV特性

- 外接信号发生器驱动
- 内置信号扫描

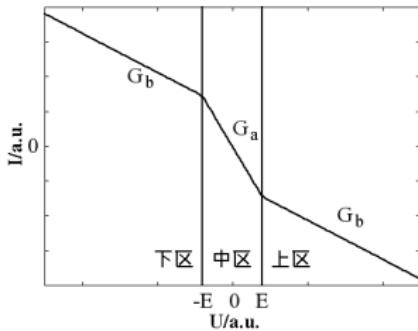


Fig: 非线性负阻IV特性示意图

外接信号发生器驱动

- 方法一：外接信号发生器扫描

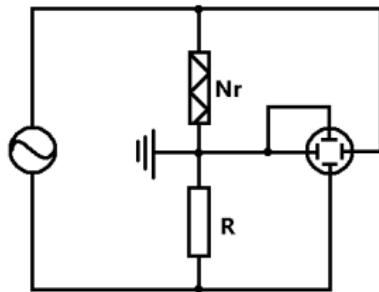
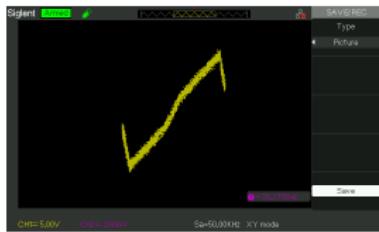


Fig: 外接信号扫描电路图

● I-V特性

100Ω, 30Hz



100Ω, 300Hz



100Ω, 3.3kHz



1000Ω, 30Hz



1000Ω, 300Hz



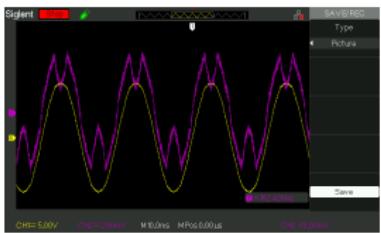
1000Ω, 3.3kHz



外接信号发生器驱动

- 频率特性

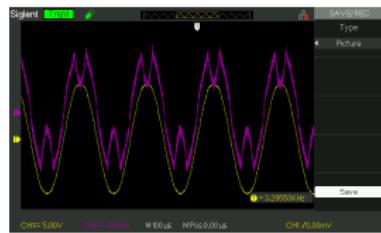
$100\Omega, 30\text{Hz}$



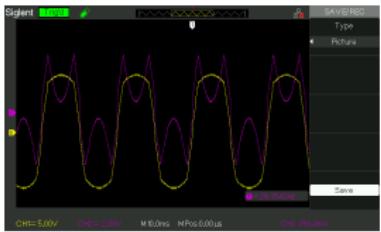
100Ω , 300Hz



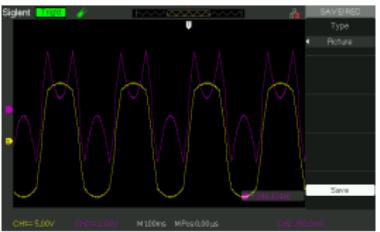
100Ω, 3.3kHz



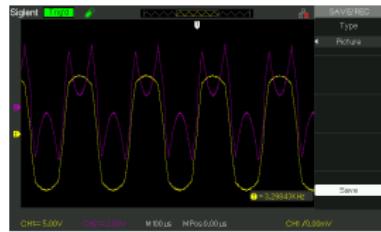
$1000\Omega, 30\text{Hz}$



1000Ω , 300Hz



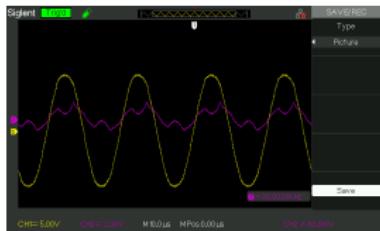
1000Ω,3.3kHz



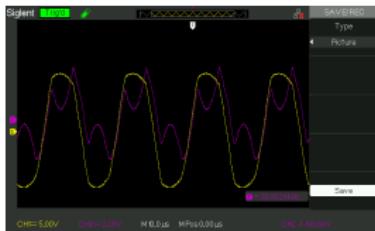
外接信号发生器驱动

- 固定外部信号频率，改变阻抗

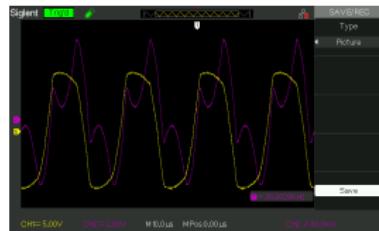
200 Ω , 30kHz



700 Ω , 30kHz



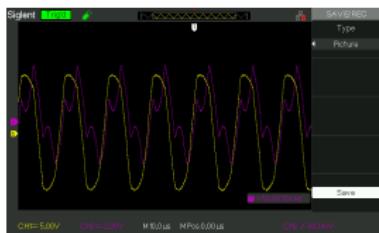
1200 Ω , 30kHz



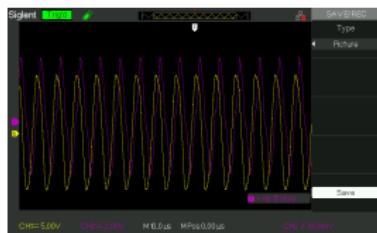
外接信号发生器驱动

- 固定阻抗，改变外部信号频率

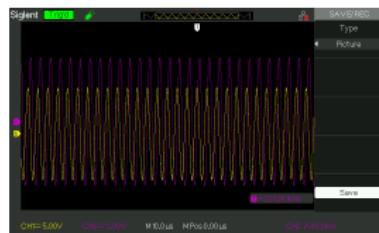
700Ω, 50kHz



700Ω, 110kHz



700Ω, 220kHz



内置信号扫描

- 方法二：内置信号扫描
- 电路中接入 100Ω 电阻，通过测量该电阻两端电压得到电流 I

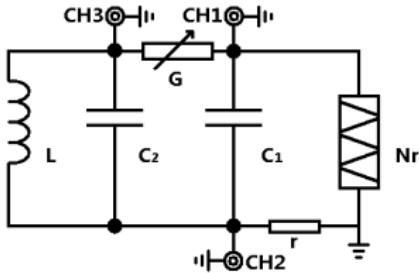
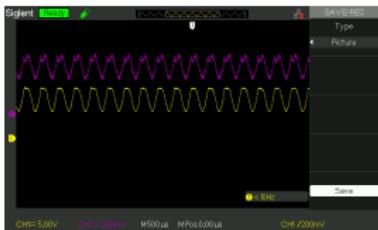


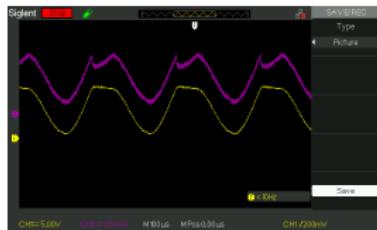
Fig: 内置信号扫描电路图

内置信号扫描

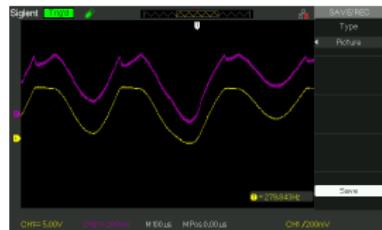
趋于不动点



单周期

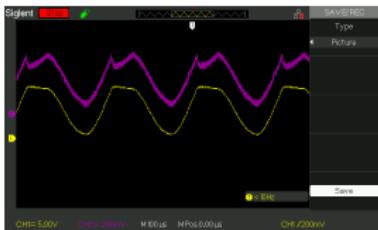


阵发混沌

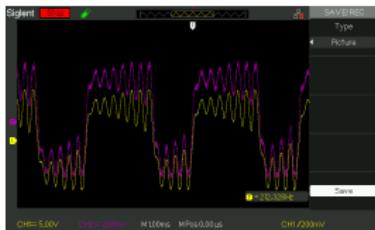


内置信号扫描

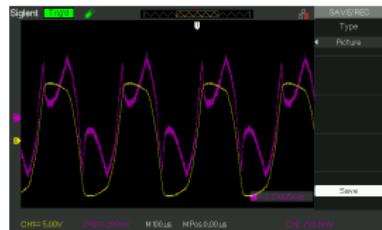
单吸引子



双吸引子

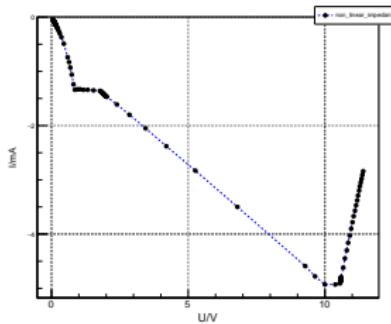
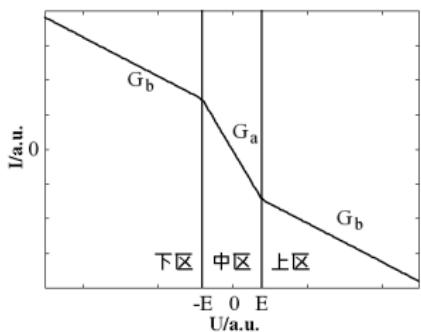


初始态



元件参数测量

- 伏安法
- $E = 1.771V$
- 仪器参数: $G_a = -7.6 \times 10^{-4} \Omega^{-1}$ $G_b = -4.09 \times 10^{-4} \Omega^{-1}$



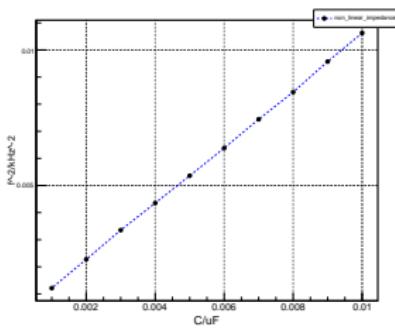
非线性负阻伏安特性曲线

非线性负阻伏安特性曲线实验结果

元件参数测量

- 电容测量：万用表
- $C_1 = 0.105\mu F \quad C_2 = 10.5nF$

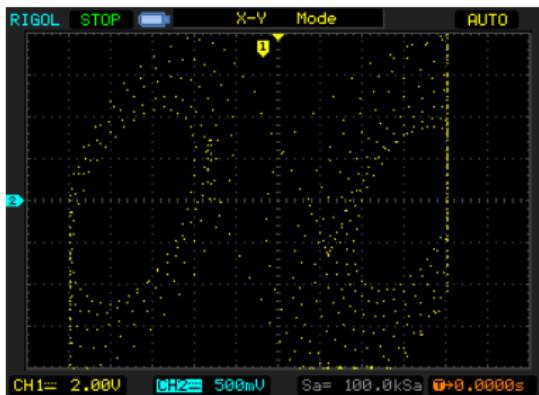
串联谐振法测量电感



$$L = \frac{1}{(2\pi f)^2 C} = (2.44 \pm 0.01) \times 10^{-2} \text{ H}$$

非线性方程求解

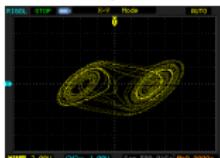
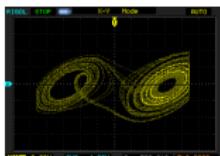
- 不动点间距测量 $1/G = R = 2000\Omega$
- $G_a = -7.6 \times 10^{-4}\Omega^{-1}$ $G_b = -4.09 \times 10^{-4}\Omega^{-1}$ $E = 1.771V$
- $\Delta U_x = 2kE = 2\frac{b-a}{b+1}E = 2\frac{G_b-G_a}{G_b+G}E = 13.66V$
- 实验值: $\Delta U_x = 6.6 \times 5.0 = 13.2V$



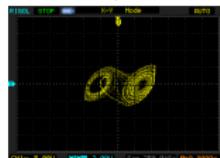
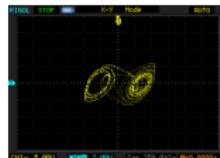
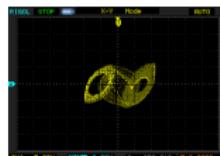
调制模式对图像大小影响

$$\bullet \Delta U_x = 2 \frac{G_b - G_a}{G_b + G} E$$

G增大

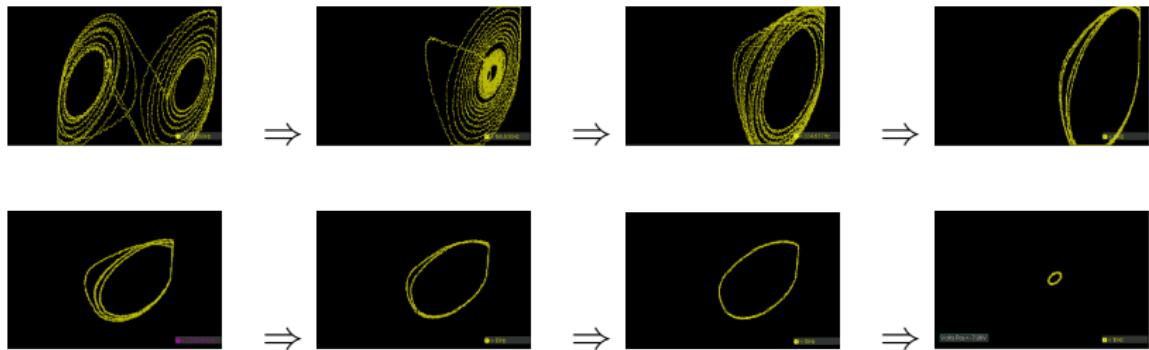


C1增大



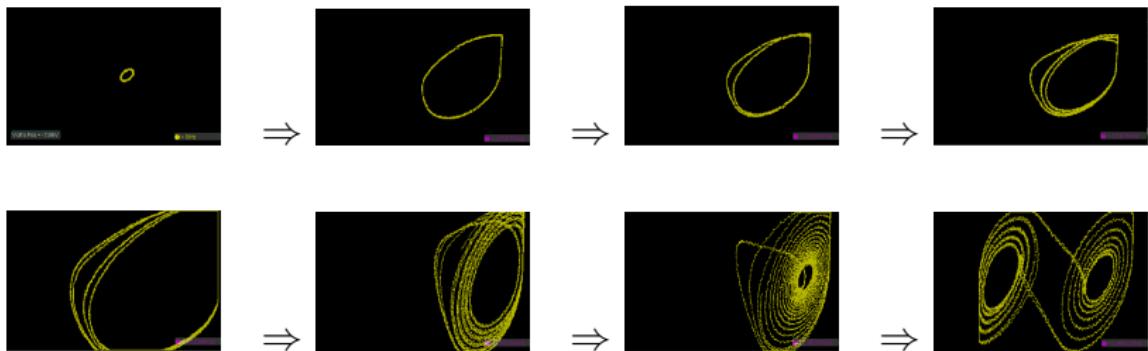
C调制

- 调节(增大) C_1



C调制

- 调节(增大) C_2



- 加密：在原始信号基础上叠加一个加密信号
- 解码：从经过处理的信号中消除加密信号
- 加密信号：混沌信号-长周期不可预测
- 条件：加密解密信号要求在同步状态，且信幅一致

- 加密：在原始信号基础上叠加一个加密信号
- 解码：从经过处理的信号中消除加密信号
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原始信号

→

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原始信号 → 加密 →

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原始信号 → 加密 →
 U_{in} →

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- 解码：从经过处理的信号中消除加密信号
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原始信号 → 加密 →
 U_{in} → $U_{in} + U_{chaos_1}$ →

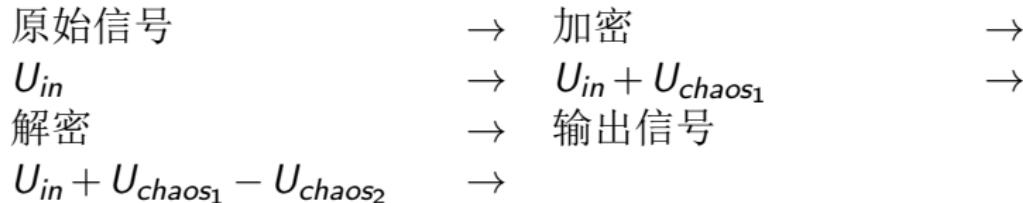
- 加密：在原始信号基础上叠加一个加密信号
- 解码：从经过处理的信号中消除加密信号
- 加密信号：混沌信号-长周期不可预测
- 条件：加密解密信号要求在同步状态，且信幅一致

原始信号 → 加密 →
 U_{in} → $U_{in} + U_{chaos_1}$ →
解密 →

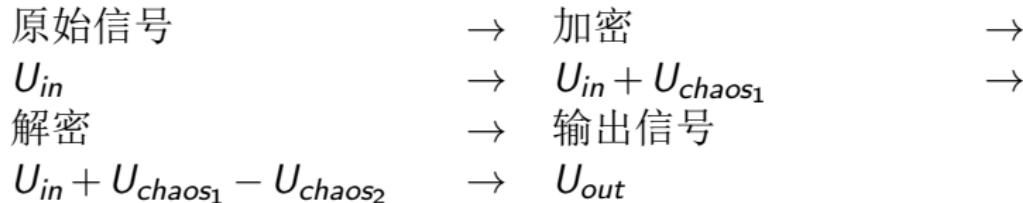
- 加密：在原始信号基础上叠加一个加密信号
- 解码：从经过处理的信号中消除加密信号
- 加密信号：混沌信号-长周期不可预测
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- 加密信号：混沌信号-长周期不可预测
- 条件：加密解密信号要求在同步状态，且信幅一致



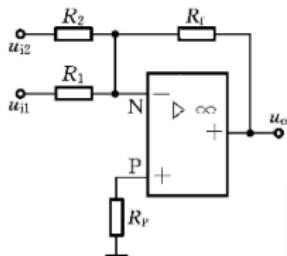
- 加密：在原始信号基础上叠加一个加密信号
- 解码：从经过处理的信号中消除加密信号
- 加密信号：混沌信号-长周期不可预测
- 条件：加密解密信号要求在同步状态，且信幅一致

原始信号	\rightarrow	加密	\rightarrow
U_{in}	\rightarrow	$U_{in} + U_{chaos_1}$	\rightarrow
解密	\rightarrow	输出信号	
$U_{in} + U_{chaos_1} - U_{chaos_2}$	\rightarrow	U_{out}	

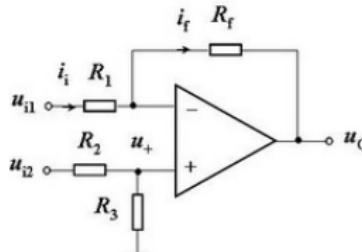
条件：两混沌信号同步 $U_{chaos_1}(t) = U_{chaos_2}(t)$

信号处理器件

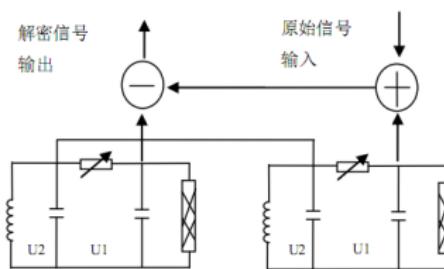
- 叠加(消除)信号方法：加(减)法器



加法器



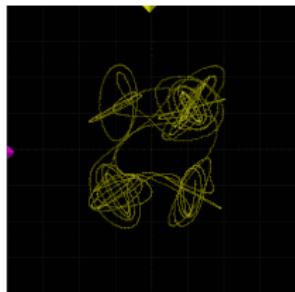
减法器



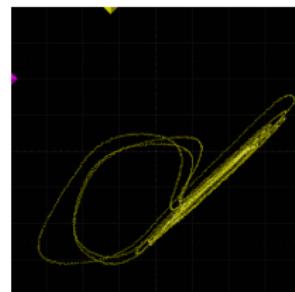
混沌通讯电路示意图

混沌信号同步

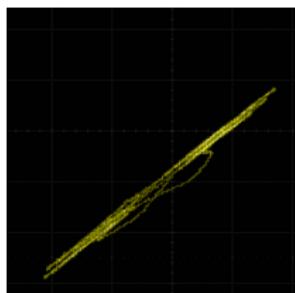
- 同步：相位、幅值



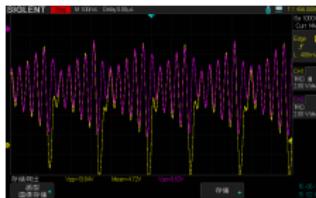
未同步



接近同步



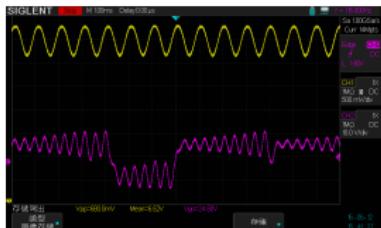
同步



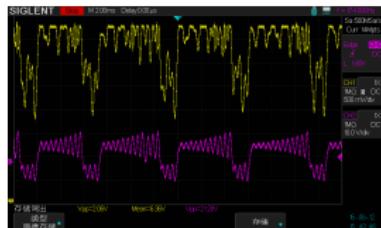
同步频率特征

混沌加密

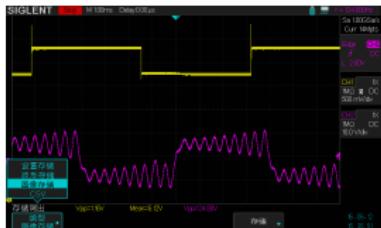
- 加密：在原始信号基础上施加一个混沌信号
- $U_{in} \rightarrow U_{in} + U_{chaos_1}$



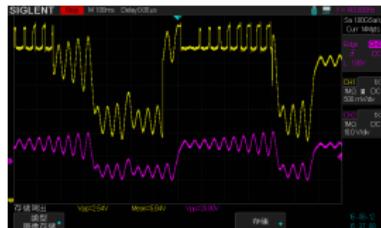
原始正弦信号



加密正弦信号



原始方波信号



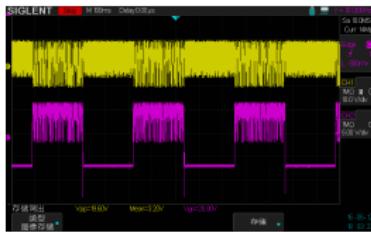
加密方波信号

混沌解码

- 解码：在经过处理的传输信号上去除一个同步的混沌信号
- $U_{in} + U_{chaos_1} - U_{chaos_2} \rightarrow U_{out}$



解码方波信号
相位差异



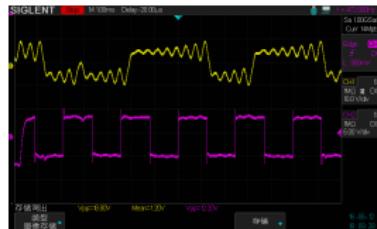
解码方波信号
幅值差异

混沌解码

- 混沌通讯：输入信号经过混沌加密处理后再经过混沌解码输出
- $U_{in} \approx U_{out}$
- $U_{chaos_1}(t) = U_{chaos_2}(t)$



输出正弦信号



输出方波信号



输出三角波信号

- 1 近代物理实验补充讲义、实验室网站
- 2 郝柏林. 混沌开创新科学[M]. 北京: 高等教育出版社, 2004
- 3 Runge-Kutta method wikipedia

- 感谢乐永康老师和蔡群老师的指导！
- 感谢陈子涵同学对加减法器的实现！

Thanks!