转动惯量不确定度计算公式:

1. 弹性系数 ***K***

[1] **Determination of *K*.**

From $T_0 = 2\pi\sqrt{I_0 / K}$ **&** $T_1 = 2\pi\sqrt{I_{PC}+I_0 / K}$

We get $T_0^2 = 4\pi^2 \dfrac{I_0}{K}$ **&** $T_1^2 = 4\pi^2 \dfrac{I_{PC}+I_0}{K}$

Thus $K = 4\pi^2 \dfrac{I_{PC}}{T_1^2 - T_0^2}$

[2] Uncertainty of *K*.

$$u(K) = \sqrt{\left[\frac{u(I_{PC})}{I_{PC}}\right]^2 + \left[\frac{u(T_1^2 - T_0^2)}{T_1^2 - T_0^2}\right]^2} \times K.$$

Please note:

$u(T_1^2 - T_0^2) = \sqrt{[u(T_1^2)]^2 + [u(T_0^2)]^2}$ **&** $u(T_1^2) = 2T_1 \times u(T_1)$.

Since we measure the time of 10 periods, so for each period, we have $T_1 = \dfrac{1}{10} t_1$ **and**

$$u(t_1) = \sqrt{u_A^2(t_1) + u_{B2}^2(t_1)}$$

$u(T_1) = \dfrac{1}{10} u(t_1)$. **Where,** $u_A(t_1) = \sqrt{\dfrac{\sum (t_{1i} - \overline{t}_1)^2}{n(n-1)}}$.

$$u_{B2}(t_1) = \frac{a}{\sqrt{3}} = \frac{0.01}{\sqrt{3}} s$$

2. Uncertainty of different objects:

[1] Plastic cylinder:

$$I_{PC} = \frac{1}{2} m r^2 = \frac{1}{8} m D^2 \quad \& \quad u(I_{PC}) = \sqrt{\left[\frac{u(m)}{m}\right]^2 + \left[2 \times \frac{u(\mathrm{D})}{D}\right]^2} \times I_{PC}$$

Where,

$$u(m) = \sqrt{u_{B1}^2(m) + u_{B2}^2(m)}$$

$$u_{B1}(m) = d = 0.1g$$

$$u_{B2}(m) = \frac{a}{\sqrt{3}} = \frac{0.2}{\sqrt{3}} g$$

&

$$u(D) = \sqrt{u_A^2(D) + u_{B2}^2(D)}$$

$$u_A(D) = \sqrt{\frac{\sum (D_i - \overline{D})^2}{n(n-1)}}.$$

$$u_{B2}(D) = \frac{a}{\sqrt{3}} = \frac{0.002}{\sqrt{3}} cm$$

[2] Metal barrel:

$$I_{MB} = \frac{K(T_1^2 - T_0^2)}{4\pi^2} \quad \& \quad u(I_{MB}) = \sqrt{\left[\frac{u(K)}{K}\right]^2 + \left[\frac{u(T_1^2 - T_0^2)}{T_1^2 - T_0^2}\right]^2} I_{MB}.$$

$$I_t = \frac{1}{8} m(D_1^2 + D_2^2) \quad \& \quad \begin{aligned} u(I_t) &= \sqrt{\left[\frac{u(m)}{m}\right]^2 + \left[\frac{u(D_1^2 + D_2^2)}{D_1^2 + D_2^2}\right]^2} I_t \\ &= \sqrt{\left[\frac{u(m)}{m}\right]^2 + \frac{\left(2D_1 u(D_1)\right)^2 + \left(2D_1 u(D_1)\right)^2}{\left(D_1^2 + D_2^2\right)^2}} I_t \end{aligned}$$

[3] Iron Ball

$$I_{IB} = \frac{K(T_1^2 - T_0^2)}{4\pi^2} \quad \& \quad u(I_{IB}) = \sqrt{\left[\frac{u(K)}{K}\right]^2 + \left[\frac{u(T_1^2 - T_0^2)}{T_1^2 - T_0^2}\right]^2} I_{IB}.$$

$$I_t = \frac{2}{5} mr^2 = \frac{1}{10} mD^2 \quad \& \quad u(I_t) = \sqrt{\left[\frac{u(m)}{m}\right]^2 + \left[2 \times \frac{u(\mathrm{D})}{D}\right]^2} \times I_t.$$

[4] Thin Bar

$$I_{TB} = \frac{K(T_1^2 - T_0^2)}{4\pi^2} \quad \& \quad u(I_{TB}) = \sqrt{\left[\frac{u(K)}{K}\right]^2 + \left[\frac{u(T_1^2 - T_0^2)}{T_1^2 - T_0^2}\right]^2} I_{TB}.$$

$$I_t = \frac{1}{12} mL^2 \quad \& \quad u(I_t) = \sqrt{\left[\frac{u(m)}{m}\right]^2 + \left[2 \times \frac{u(L)}{L}\right]^2} \times I_t.$$