3.5 TIME DISTRIBUTION OF NUCLEAR RADIATION[1]

It has already been mentioned that nuclear radiation and cosmic rays are random processes and therefore follow a Poisson distribution. One consequence of this distribution is that when a large number of counts, N, is considered, the distribution becomes Gaussian, with a standard deviation $\sigma = \sqrt{N}$. To test the Gnussian nature of the frequency distribution of counts, we may take several measurements of the same source and plot the results. Such data are shown in Fig. 5.23a, where the Gaussian curve, normalized to the same area as the histogram and with the appropriate σ, has also been plotted; the fit is quite satisfactory (that is, it has a low χ^2 probability as explained in Chapter 10).

The data for this plot were taken from 100 repeated measurements of approximately 100 counts each; the parameters of the sample are

$$\text{mean} \quad \bar{N} = 85.34 \text{ counts /min}$$
$$\text{standard deviation} \quad \sigma = \sqrt{N} = 9.23 \text{ counts /min} \tag{3.6}$$

Table 5.2 gives the frequency distribution of these 100 measurements in "bins" of a width of 2 counts /min. Note that for the histogram of Fig. 5.23a, the bin size was taken twice as large.

The procedure for testing if the distribution of a sample, as in Table 5.2 above, is indeed Gaussian may be facilitated by use of a special graphing technique shown in Fig. 5.23b. Here probability paper[2] is used, where on the abscissa is given the integrated probability of obtaining a count smaller than the value shown on the ordinate.

The advantage of this plot is that a Gaussian distribution appears as a straight line with a slope determined by the standard deviation. Indeed this is the case in Fig. 5.23b, where the same data of Table 5.2 have been plotted; the line is the prediction for a Gaussian distribution with a mean and standard deviation as obtained from the sample (Eq. 3.6).

We know, however, that the frequency distribution of nuclear radiation is of the Poisson type[3], which in the limit of many counts tends towards the Gaussian. To test the applicability of the Poisson distribution, we have to examine parameters which are related to few counts, for example, the distribution of the time intervals between any two successive counts (namely, the time elapsed after the arrival of one count until the next one). Similarly we may test the distribution of the time intervals from any count to the second next, and so on; however, as the number of counts contained in the time intervals is increased, the distribution tends more and more to a Gaussian. These frequency distributions have been calculated in Section 5 of Chapter 10; the result is

$$q_m(t) = r\frac{(rt)^{m-1}e^{-rt}}{(\mathrm{m}-1)!} \tag{3.7}$$

where $q_m(t)$ gives the probability of finding a time interval between t and t + dt; r is the true

1. To follow the content of this section the student must understand the material of par. 3, Chapter 10, and especially Section 5.

2. O. Riedel, "Statistical Purity in Nuclear Counting," *Nucleonics*, 12, 64 (1954).

3. Chapter 10, Section 5.

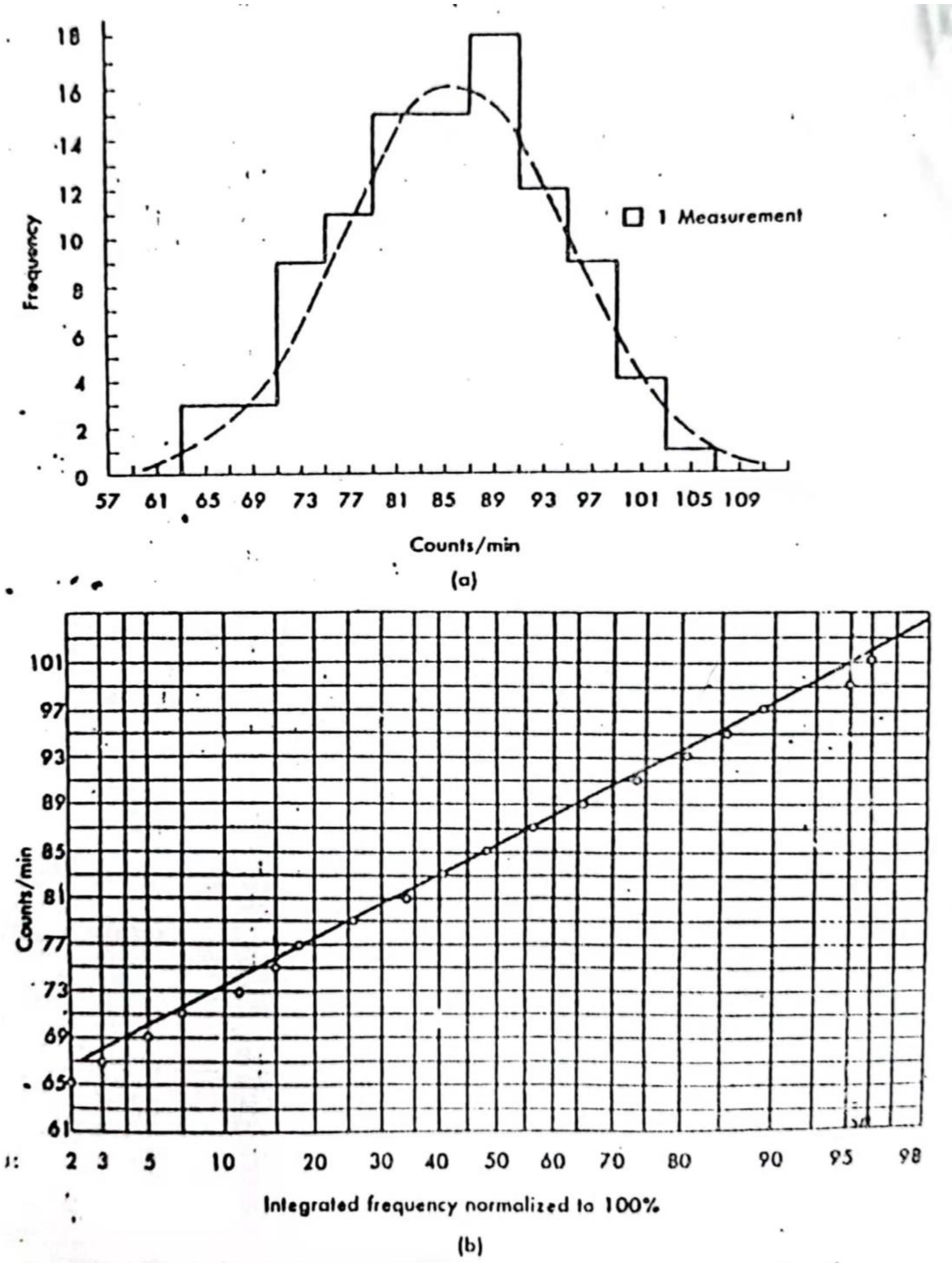

Fig. 5.23 Frequency distribution of 100 repeated measurements of a radioactive sample. (a) Histogram of the measurements; the dotted curve is the best fit Gaussian. (b)The integrated frequency distribution plotted on a "probability scale"; in such a representation a Gaussian curve appears as a straight line.

TABLE 5.2

FREQUENCY DISTRIBUTION OF 100 REPEATED MEASUREMENTS OF A RADIOACTIVE SAMPLE

Interval (counts/min)	Number of measurements (yielding a result in this interval)	Cumulative % probability (of obtaining this result)
64–65	2	2
66–67	1	3
68–69	2	5
70–71	1	6
72–73	6	12
74–75	3	15
76–77	3	18
78–79	8	26
80–81	8	34
82–83	7	41
84–85	8	49
86–87	7	56
88–89	11	67
90–91	7	74
92–93	8	82
94–95	4	86
96–97	3	89
98–99	6	95
100–101	1	96
102–103	3	99
104–105	1	100

mean rate of counts (that is, counts per unit time, where the same are to be used for measuring t), and m is the number of counts between which the time intervals are measured. If we measure the intervals between every count, m =1; if between every second count, m = 2, etc.

We note that the distribution for m = 1 is simply

$$q_1(t) = re^{-rt} \tag{3.8}$$

it has no maximum and is peaked at t = 0. This is true for any random process, and seems to justify an old German proverb that one calamity is always followed by a second one.[1] For $m \neq 1$, however, Eq. 3.7 has a maximum at

$$t = \frac{m-1}{r}$$

which for large m becomes more pronounced and also approaches the mean time interval between m counts, m/r.

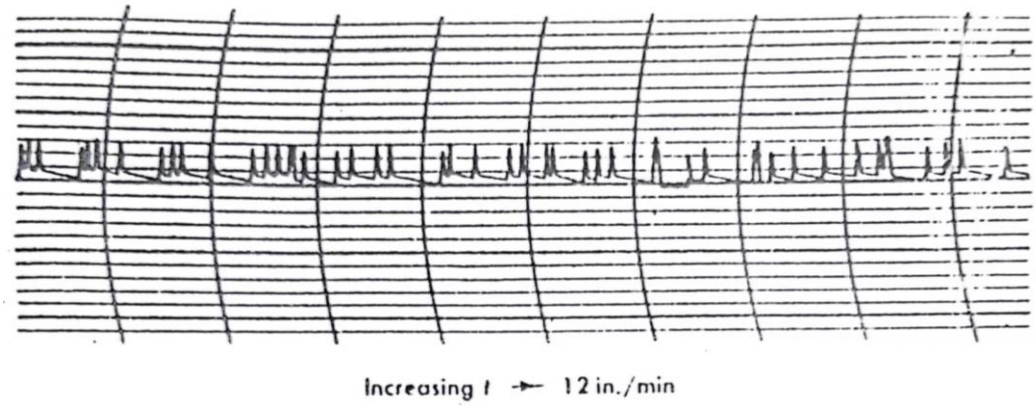

Fig. 5.24 Reproduction of the tape of the event recorder indicating the arrival of single cosmic-ray counts; the time scale is 5 sec /in.

To obtain experimental data on such distributions of nuclear radiation, the output of the Geiger-counter amplifier is connected to a pen recorder (Esterline Angus-Model AW). It is convenient to use the cosmic rays as a source of radiation, in which case a recorder paper speed of 12 in/min provides adequate spacing between counts; Fig. 5.24 is a reproduction of such a record.

All required information is then contained on the recording, and we may measure off (as with a ruler) the time intervals between each count. The same recording may be used for time intervals between second or tenth count, etc., but this becomes tedious. If such distributions are desired, it is preferable to connect the recorder to the second, or tenth stage, etc., of a scaler which is driven by the Geiger counter.

In Fig. 5.25 are presented data obtained in this fashion by students.[2] Figure 5.25a gives the histogram of the frequency distribution of the intervals between every count; the solid curve is Eq. 3.8 properly normalized to equal areas and indicates a very satisfactory fit. The histogram in Fig. 5.25b is the frequency distribution of the time intervals between every second count for the same sample; again the solid curve is obtained from Eq. 3.7 with $m = 2$, and also shows a satisfactory fit.

1. W. Bothe, Physikalishe Zeitschrift, 37, 520(1936).

2. D. Owen and D. Sawyer, class of 1 963.

Distribution of time intervals between every count

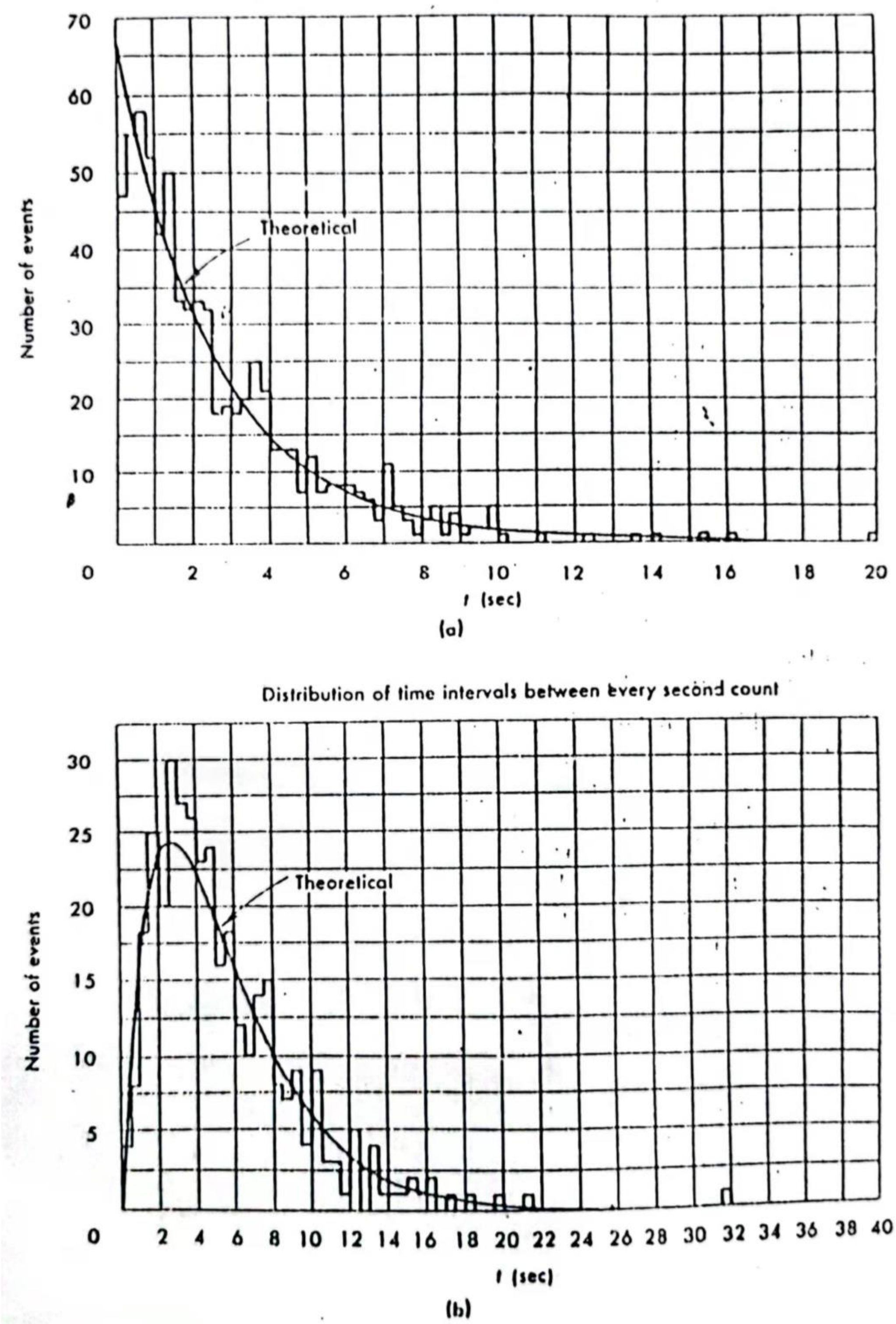

Fig. 5.25. The distribution of time intervals between the arrival of cosmic-ray counts as obtained from the record of Fig. 5.24. The solid curves are the predictions of Eq. 3.7. (a) Time intervals between every count (*m* =1). (b) Time intervals between every second count (*m* = 2).