ACKNOWLEDGMENT

G. B. Shoop participated in the development of the source deposition technique and performed the actual assembly of the batteries. J. Scott-Monck made the gallium arsenide diodes and some of the silicon diodes. E. F. Pasierb, Jr., made the later silicon diodes. R. C. Hand designed the atomic battery assembly. P. Rappaport, who first conceived of this method of energy conversion, has provided helpful discussion.

TABLE OF SYMBOLS

$A, A'$ = Parameters used to describe the rate of degradation of semiconductors under electron irradiation.
$\alpha_0$ = Approximate self-absorption coefficient of $Pm^{147}$ beta rays in $cm^{-1}$.
$\alpha/\rho$ = Approximate absorption coefficient of $Pm^{147}$ beta rays in $cm^2/gm$.
$B$ = Empirical constant to describe current voltage relationship of a diode.
$E_B$ = Energy of an emitted electron.
$E_{B_{av}}$ = Average energy of betas from a radioisotope.
$E_{B_{max}}$ = Maximum energy of betas from a radioisotope.
$E_g$ = Energy gap of a semiconductor.
$G$ = Specific activity of a radioisotope in curies/gm.
$I_B$ = Electron current incident on an electron-voltaic converter.
$I_L$ = Current through the load of an electron voltaic converter.
$I_{max}$ = The self-absorption limited electron current which would be emitted from the surface of a radioactive layer of semi-infinite thickness.
$I_0$ = Reverse saturation current of a diode.
$I_s$ = Short-circuit current of an electron-voltaic converter.
$k$ = Boltzmann's constant.
$L_n$ = Diffusion length of electrons.
$L_p$ = Diffusion length of holes.
$\lambda$ = $q/kT$.
$\eta_{max}$ = Electron-voltaic conversion efficiency.
$q$ = Electronic charge.
$t_H$ = Radioisotope half-life.
$\tau$ = Minority carrier lifetime.
$\tau_0$ = Minority carrier lifetime before electron irradiation.
$V_{max}$ = Voltage across an electron-voltaic converter under open circuit conditions.
$V_{mp}$ = Voltage across an electron-voltaic converter at a load which gives maximum power.
$w$ = Ionization energy required to generate and collect an electron-hole pair in a diode.

# Analysis and Characterization of *P-N* Junction Diode Switching*

H. J. KUNO†, MEMBER, IEEE

***Summary*—A new charge control model of a *p-n* junction diode is introduced in which the reverse current $i_R$ as well as the forward current $I_F$ are related to the charge $Q$ stored in the base region by time constants $\tau_R$ and $\tau_F$, respectively. The reverse switching transient is analyzed for normal switching operation where a constant current phase (storage phase) and a decaying current phase exist, and for overdriven switching operation where no constant current phase exists.**

**New switching time equations are derived. The equations are expressed in terms of measurable device parameters $\tau_F$, $\tau_R$, and $C_j$; external circuit variables $I_F$ and $I_R$; and an external circuit parameter $R$. The proposed model is applicable to *p-n* junction diodes of any type.**

**Experimental results using various types of diodes are also reported. It is shown that the experimental results are in very good agreement with the theory.**

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## INTRODUCTION

THE SWITCHING TIME of a semiconductor diode is of great importance in computer circuit design. Recently, in particular, development of very high speed switching transistors made the switching time of diodes much more significant than ever before. The switching time analysis of diodes have been carried out by some authors [1]–[8]. However, the switching time equations derived by most of the authors in the past, [1]–[4], [7], [8], are expressed in terms of parameters which are extremely difficult to measure and not practical for characterizing the diode switching. It is, therefore, desirable for practical circuit design purposes to find new ways of characterizing the diode switching in terms of parameters which can be measured simply.

In this paper, the reverse switching transient is analyzed and new diode switching time equations are derived by relating the reverse current $i_R$ as well as the forward

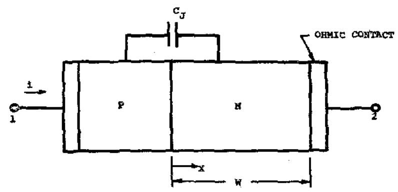


Fig. 1—Planar junction diode model.

current $I_F$ to the minority carrier charge $Q$ stored in the base region of the diode by the reverse and forward time constants $\tau_R$ and $\tau_F$, respectively. The equations are expressed in terms of measurable diode parameters, *viz.*, $\tau_F$, $\tau_R$, and $C_j$, external circuit resistor $R$, and the forward to reverse current ratio $I_F/I_R$.

## Analysis of Switching Operation

### *Charge Equation*

In order to simplify the analysis, let us consider a planar junction diode shown in Fig. 1. (We do not lose generality by this simplification.) We shall also assume that the conductivity of the $p$-type material $\sigma_p$ is much greater than that of the $n$-type material $\sigma_n$, so that the hole current at the junction may be considered to be the total current. The continuity equation in the $n$-type (base) region can be written in terms of the excess hole density as [8]

$$q\frac{\partial p}{\partial t} = -\frac{\partial i_p}{\partial x} - q\frac{p}{\tau_p}$$

where

$$p(x, t) = P(x, t) - P_{0(x)}$$

$$i_p = q\left(\mu_p EP - D_p\frac{\partial P}{\partial x}\right)$$

= hole current
$P(x, t)$ = hole density
$P_0(x)$ = hole density in thermal equilibrium.

Integrating this equation over the region $0 \leq x \leq w$, and defining the total charge stored in the base region $Q$ such that

$$Q(t) = q\int_0^w p\,dx,$$

we get

$$\frac{dQ}{dt} = i_p(0, t) - i_p(w, t) - \frac{Q}{\tau_p}.$$

Noting that the term $Q/\tau_p$ represents the recombination rate in the base region where $0 < x < w$ and that $i_p(w, t)$ represents the recombination rate at the boundary $x = w$, let us define the total recombination rate constant, $\tau_F$,[1] such that

$$i_p(w, t) + \frac{Q(t)}{\tau_p} = \frac{Q(t)}{\tau_F}.$$

Then we get

$$\frac{dQ(t)}{dt} = i_p(0, t) - \frac{Q(t)}{\tau_F}.$$

Considering the junction capacitance $C_j$ as shown in Fig. 1, the total current $i(t)$ flowing into the terminal 1 is given by

$$i(t) = i_p(0, t) + \frac{d(C_jV_j)}{dt}$$

$$= i_p(0, t) + \bar{C}_j\frac{dV_j}{dt}$$

where

$V_j$ = junction voltage

$$\bar{C}_j = \frac{1}{\Delta V_j}\int C_j(V_j)\,dV_j.$$

Thus we can relate the total current $i(t)$ that flows into the terminal 1 to the charge $Q(t)$ stored in the base region of the diode and the junction voltage $V_j$ by the following, namely, the charge equation

$$\boxed{i(t) = \frac{dQ}{dt} + \frac{Q}{\tau_F} + C_j\frac{dV_j}{dt}}. \tag{1}$$

### *Initial Condition*

Now let us consider the switching circuit shown in Fig. 2. Initially, the switch $S$ is in position 1, and the diode is forwardly biased and conducting current $I_F$. (We shall assume that the switch $S$ has been in position 1 for a long time so that a steady state may be assumed at the time $t = 0^-$). Then the initial condition can be obtained by substituting steady-state conditions

$$i(0^-) = I_F$$

$$\frac{dQ}{dt} = 0$$

$$\frac{dV_j}{dt} = 0$$

into (1).

$$Q(0) = I_F\tau_F. \tag{2}$$

[1] The subscript $F$ is chosen since, for steady state, $Q$ is related to $I_F$ by

$$Q = I_F\tau_F.$$

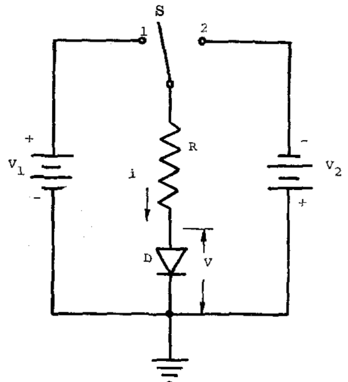


Fig. 2—Diode switching circuit.

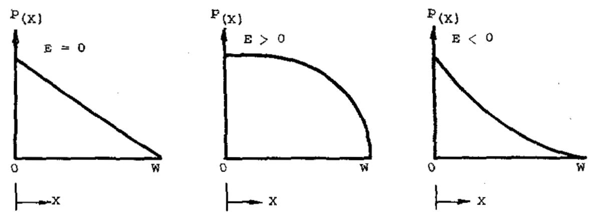


Fig. 3—Steady state hole density distributions in the base region of various types of diodes.

The initial hole density distribution can be found by solving the diffusion equation. Typical steady-state distributions are shown in Fig. 3.

*Normal Switching Operation*

In normal switching operation, switching transient consists of constant current phase (storage phase) and decaying current phase as shown in Fig. 4. Now suppose that at $t = 0^+$ the switch $S$ is quickly thrown from position 1 to position 2. Then the current starts to flow in the opposite direction. The maximum reverse current is limited, by the external resistor $R$, to

$$I_R = \frac{V_2 - V_j}{R} \cong \frac{V_2}{R}. \tag{3}$$

Then the stored charge $Q$ starts to decay in a manner shown in Fig. 5.

For $0 < t < t_s$, (see Fig. 4), the constant reverse current $I_R$ flows and the junction voltage does not change. (Strictly speaking, it changes slightly as shown in Fig. 4, but the term $C_j(dV_j/dt)$ is negligible compared with other terms in (1) during the constant current phase.) Then (1) reduces to

$$-I_R = \frac{dQ}{dt} + \frac{Q}{\tau_F}. \tag{4}$$

With the initial condition given by (2), we can solve for $Q(t)$ and get

$$Q(t) = (I_F + I_R)\tau_F \exp\left(-\frac{t}{\tau_F}\right) - I_R\tau_F. \tag{5}$$

During the constant current phase, *i.e.*, $0 < t < t_s$, the reverse current $I_R$ is given by

$$-I_R = q\left[\mu EP - D_p \frac{\partial P}{\partial x}\right]_{x=0} \quad \text{for} \quad 0 \leq t < t_s. \tag{6}$$

At $t = t_s$, the hole density at the junction ($x = 0$) becomes zero, and (6) becomes

$$-I_R = -qD_p \frac{\partial P}{\partial x}\bigg|_{x=0} \quad \text{at} \quad t = t_s. \tag{7}$$

At this time, the total charge in the base region is given by[2]

$$Q(t_s) = q\int_0^W P(x, t_s)\,dx \quad \text{at} \quad t = t_s.$$

Noting that $Q(t_s)$ is related to $I_R$, let us define $\tau_R$, relating the minimum amount of charge remaining in the base region which is required to support the reverse current, such that

$$Q(t_s) = I_R\tau_R. \tag{8}$$

Although it is not immediately obvious why the minimum amount of charge required to support the reverse current is linearly related to the reverse current, the justification is made empirically as shown in a later section.

Substituting (8) into (4), and solving for the storage time $t_s$, we get

$$\boxed{t_s = \tau_F\left[\ln\left(1 + \frac{I_F}{I_R}\right) - \ln\left(1 + \frac{\tau_R}{\tau_F}\right)\right]}. \tag{9}$$

If we plot $t_s$ vs $\ln(1 + I_F/I_R)$, we will get a straight line with a slope $\tau_F$ and offset by $\ln(1 + \tau_R/\tau_F)$, as shown in Fig. 6.

For $t > t_s$, the hole density decays further, and current starts to decay, since the stored charge cannot keep the gradient at $x = 0$ necessary to support $I_R$. Let us assume that the reverse current $i_R(t)$ is related to the charge $Q(t)$, as we have defined $\tau_R$, by the relationship

$$Q(t) = \tau_R i_R(t) \tag{10}$$

where

$$t > t_s.$$

Considering the circuit shown in Fig. 2, the junction voltage $V_j$ is related to $i_R$ by

[2] LeCan *et al.*, [5], and Moll *et al.*, [6], assume that $Q(t_s) = 0$, thus obtaining $t_s = \tau_s \ln(1 + I_F/I_R)$, but their assumptions are not correct, since the current $I_R$ at $t = t_s$ is supported by the hole density gradient at the junction as given by (7).

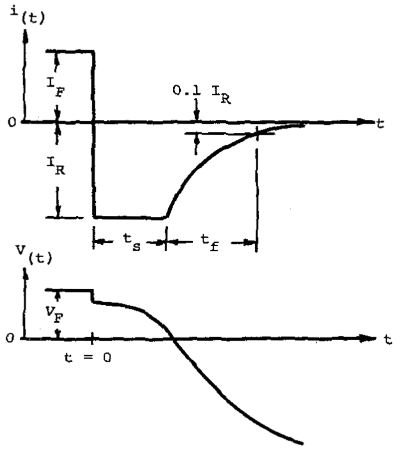


Fig. 4—Current and voltage switching characteristics of a diode.

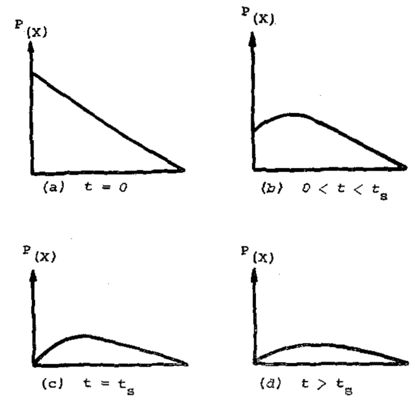


Fig. 5—Hole density distributions during the switching.

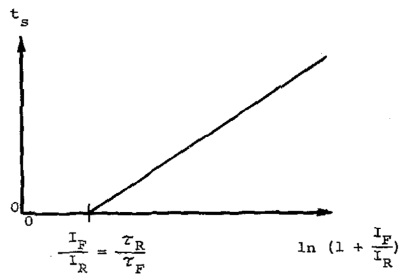


Fig. 6—Storage time $t_s$ of a diode as a function of $I_F/I_R$.

$$V_j = -V_2 + i_R R. \tag{11}$$

Differentiation of this with respect to time $t$ gives

$$\frac{dV_j}{dt} = R\frac{di_R}{dt}. \tag{12}$$

Then (1) becomes

$$-i_R = (\tau_R + RC_j)\frac{di_R}{dt} + \frac{\tau_R}{\tau_F} i_R \tag{13}$$

with the boundary condition given by (8); *i.e.*,

$$Q(t_s) = I_R \tau_R$$

we can solve for $i_R(t)$ for $t > t_s$. We get

$$i_R(t) = I_R \exp\left\{-\left[\frac{1 + \frac{\tau_R}{\tau_F}}{\tau_R + RC_j}\right](t - t_s)\right\}. \tag{14}$$

Defining $t_f$ as the time that $i_R$ takes to decay from $I_R$ to 10 per cent of $I_R$ as shown in Fig. 4, *i.e.*,

$$i_R(t_f) = 0.1\, I_R,$$

$t_f$ can be obtained from (14).

$$t_f = \left[\frac{\tau_R + RC_j}{1 + \frac{\tau_R}{\tau_F}}\right] \ln 10 \tag{15}$$

or

$$\boxed{t_f = 2.3\left[\frac{\tau_R + RC_j}{1 + \frac{\tau_R}{\tau_F}}\right]}. \tag{16}$$

### *Overdriven Switching Operation*

In normal switching operation, we have derived the expression for the storage time $t_s$ given by (9). It can be seen that, as $I_R$ is increased keeping $I_F$ constant, $t_s$ decreases and becomes zero when

$$I_R = \frac{\tau_F}{\tau_R} I_F.$$

Let us define $I_{RO}$ as the critical reverse current beyond which $t_s$ becomes zero, *i.e.*,

$$I_{RO} = \frac{\tau_F}{\tau_R} I_F. \tag{17}$$

When $i_r > I_{RO}$, *viz.*, overdriven switching, the charge stored in the base region cannot support the reverse current. Thus, the reverse current immediately starts to decay to $I_{RO}$. Let us define $t_0$ such that

$$i_r(t_0) = I_{RO}, \tag{18}$$

as shown in Fig. 7, and assume that $t_0$ is very small so that the charge stored in the base region does not decay appreciably during this interval of time.[3] Then the all

[3] This assumption is justifiable since the time constant $RC_j$, thus $t_0$, is very small compared with $\tau_F$ in most cases.

current flowing during the interval $t_0$ may be considered to discharge the junction capacitance, and for the overdriven switching, where $t \leq t_0$, (1) reduces to

$$-i_R(t) = I_F + RC_i \frac{di_R}{dt} \tag{19}$$

where

$$i_R(t) > i_R(t_0) = I_{RO}$$

$$Q(t) = I_F \tau_F = \text{constant for } t < t_0.$$

The initial condition is

$$i_R(0) = I_R' \tag{20}$$

where $I_R'$ is defined as the overdriven peak current which is determined by the external circuit such that

$$I_R' = \frac{V_2 - V_i}{R} \cong \frac{V_2}{R}, \tag{21}$$

as shown in Fig. 7.

The solution of (19) with the initial condition (20) is then given by

$$i_R(t) = (I_F + I_R') \exp\left(-\frac{t}{RC_i}\right) - I_F \tag{22}$$

for

$$i_R(t) > I_{RO},$$

*i.e.*,

$$0 < t < t_0.$$

At the end of the overdriven phase; *i.e.*, $t = t_0$, we have

$$i_R(t_0) = I_{RO}. \tag{23}$$

Substituting (23) into (22) and solving for $t_0$, we obtain

$$t_0 = RC_i\left[\ln\left(1 + \frac{I_R'}{I_F}\right) - \ln\left(1 + \frac{I_{RO}}{I_F}\right)\right] \tag{24}$$

or

$$\boxed{t_0 = RC_i\left[\ln\left(1 + \frac{I_R'}{I_F}\right) - \ln\left(1 + \frac{\tau_F}{\tau_R}\right)\right]}. \tag{25}$$

For $t > t_0$, we have $i_R < I_{RO}$ and

$$\tau_R i_R = Q(t).$$

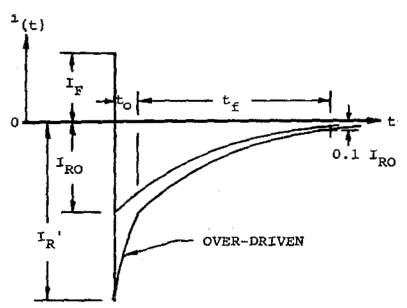


Fig. 7—Current switching characteristics of a diode overdriven in the reverse direction.

Then (1) becomes

$$i_R(t) = (\tau_R + RC_i)\frac{di_R}{dt} + \frac{\tau_R}{\tau_F} i_R \tag{26}$$

with the boundary condition

$$i_R(t_0) = I_{RO},$$

the solution of (26) becomes

$$i_R(t) = I_{RO} \exp\left[-\left[\frac{1 + \frac{\tau_R}{\tau_F}}{\tau_R + RC_i}\right](t - t_0)\right] \tag{27}$$

and

$$t_f = 2.3\left[\frac{\tau_R + RC_i}{1 + \frac{\tau_R}{\tau_F}}\right] \tag{28}$$

where $t_f$ is defined as the time required for $i_R$ to decay from $I_{RO}$ to 10 per cent of $I_{RO}$; *i.e.*,

$$i(t_f) = 0.1\, I_{RO}.$$

## Experimental Results

Various types of diodes were selected for switching time measurement. Also, current waveforms during the switching transient were analyzed using the set-up shown in Fig. 8. Diodes tested are grouped into four general types: 1) germanium gold bonded, 2) germanium expitaxial mesa, 3) silicon planar, and 4) silicon diffused mesa. Fig. 9(a)–9(d) shows typical results of the storage time measurement of each group. $I_F$ was varied from 1 ma to 20 ma and $I_F/I_R$ up to 100.

It can be seen that each curve shows straight line relationship between $t_s$ and ln $(1 + I_F/I_R)$, as predicted by (9). The results also indicate that $\tau_F$ and $\tau_R$ are very constant over the wide range of $I_F$ and $I_R$, as we have assumed.

Fig. 10(a)–10(b) shows typical switching transient characteristics observed with a sampling scope driving a $X$–$Y$ recorder. Fig. 10(a) shows normal switching operation, and Fig. 10(b) overdriven switching operation. These results show very good agreement with the switching characteristics given by (14), (22), and (27).

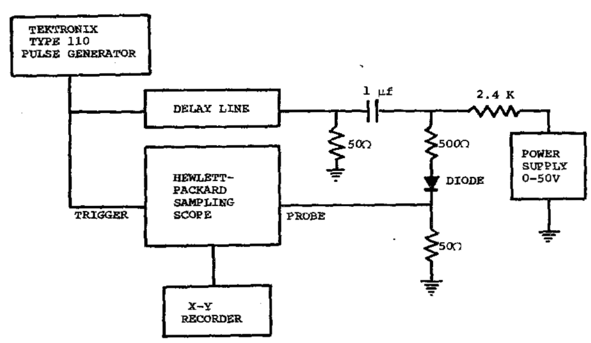


Fig. 8—Set-up for diode switching test.

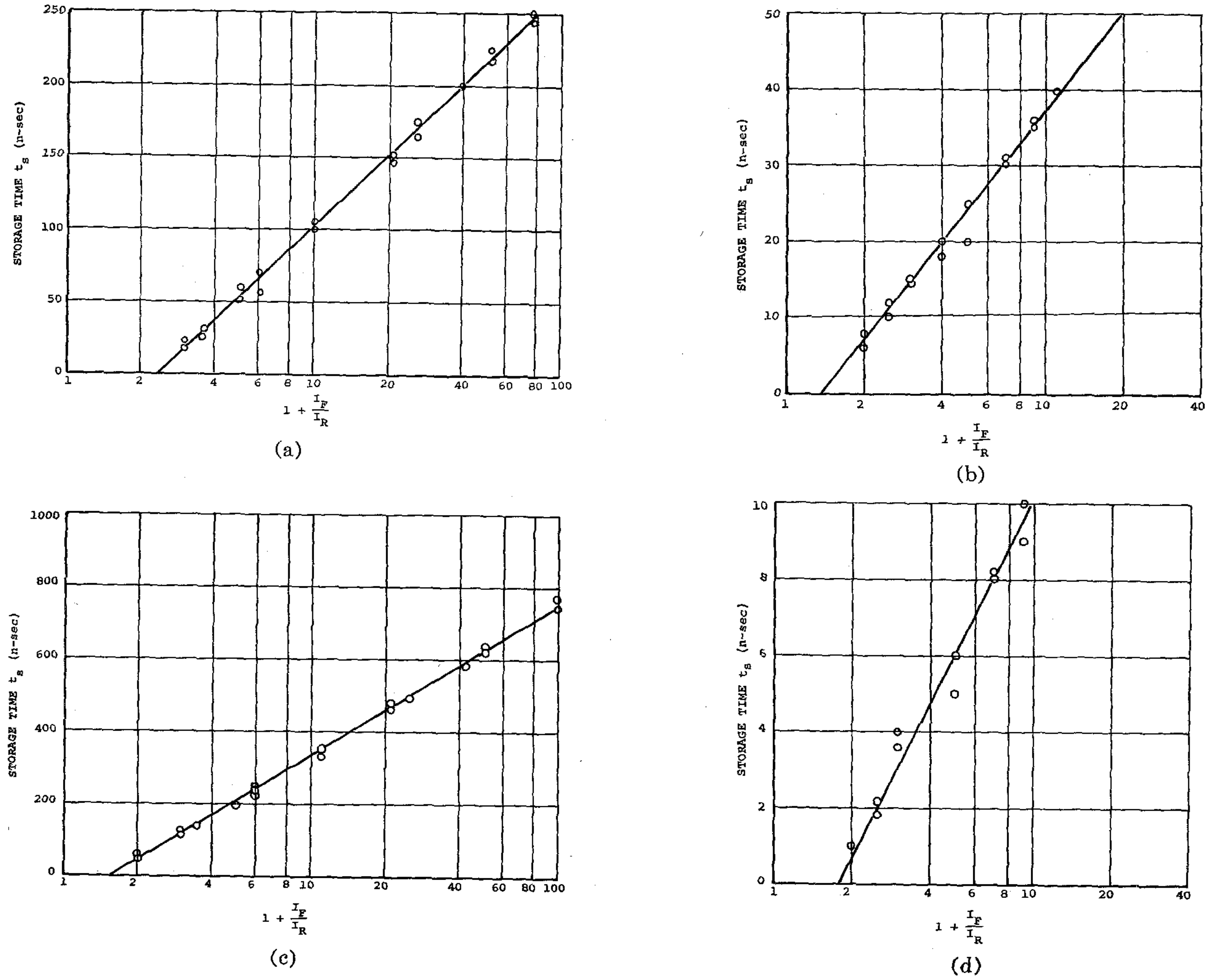


Fig. 9—Storage time as a function of $1 + I_F/I_R$ for various types of diodes. (a) Diode H24001[4] (Type 1). (b) Collector to base diode of transistor 2N964 (Type 2). (c) Diode H25000[4] (Type 3). (d) Diode 1N914 (Type 4).

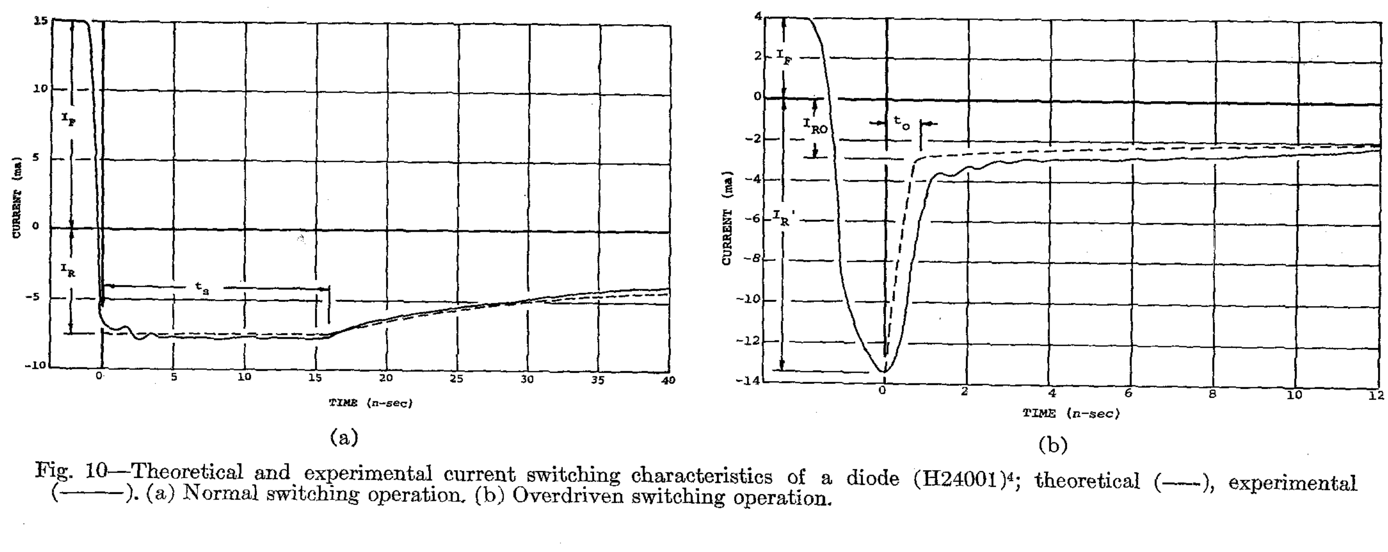


Fig. 10—Theoretical and experimental current switching characteristics of a diode (H24001)[4]; theoretical (——), experimental (———). (a) Normal switching operation. (b) Overdriven switching operation.

[4] *H numbers indicate National Cash Register Co. specification numbers.*

### Discussion

The proposed charge control model of diode assumes linear relationships between $I_F$ and $Q$, and between $i_R$ and $Q$, given by (2) and (10), respectively. The experimental results show that $\tau_F$ and $\tau_R$ are very constant; thus the assumption is justified. Analytical verification may, however, be carried out by solving the diffusion equation for $P(x, t)$ and integrating over the region $0 \leq x \leq w$, though it is a very complicated task.

The good agreement of the experimental results with the theory indicates that proposed model can characterize reverse switching transient of *p-n* junction diodes of any type by measuring $\tau_F$, $\tau_R$, and $C_j$.

The proposed model may also be applied to collector-base junction of a transistor to characterize the storage time of the transistor.

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# Impedance Characteristics of an AC MHD Generator*

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***Summary*—A study is made of those parameters, associated with the impedance of an ac MHD generator, which affect the power factor. It has been found that the addition of a ferromagnetic material in the region external to the plasma causes a significant increase in the power factor. An expression for the actual electrical power extracted from the generator, as determined by electrical impedance methods, is shown to be identical to the expression derived by considering Lorentz forces on elements of the fluid for a particular approximation mentioned in the report.**

### Introduction

THE INTERNAL and kinetic energy of a flowing plasma in the presence of a magnetic field may convert some of its energy directly into electrical form. This process of energy conversion has been investigated extensively of late and is known as "magnetohydrodynamic" power generation.

There are two basic schemes for MHD power extraction. In the first scheme, electrodes are adjacent to the plasma. These electrodes are connected to a load resistance causing a dc current to flow through the circuit, thereby delivering power to the load. This method is summarized by Steg and Sutton.[1] DC generators have the disadvantage, for certain applications, of requiring a DC–AC converter.

The second approach, which is the one considered herein, utilizes a traveling-wave type magnetic field, the power being extracted by induction. The phase velocity of the traveling magnetic field may be larger or less than the plasma velocity. In the former case, the plasma is accelerated; that is, electrical energy is converted into internal and kinetic energy. In the latter situation, the process is reversed; the behavior of this device is then analogous to an induction generator. The induction generator is, of course, ac, and electrodes and converters are not required.

The purpose of this paper is to investigate the various parameters which affect the impedance characteristics of the traveling-wave MHD power generator, the primary aim being to improve the power factor. In the paper of Bernstein *et al.*,[2] it was noted that for a generator which will deliver a sizeable power, the power factor is ex-

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