

Irradiation of an Accretion Disc by a Jet: General Properties and Implications for Spin Measurements of Black Holes

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ABSTRACT

X-ray irradiation of the accretion disc leads to strong reflection features, which are then broadened and distorted by relativistic effects. We present a detailed, general relativistic approach to model this irradiation for different geometries of the primary X-ray source. These geometries include the standard point source on the rotational axis as well as more jet-like sources, which are radially elongated and accelerating. Incorporating this code in the RELLINE model for relativistic line emission, the line shape for any configuration can be predicted. We study how different irradiation geometries affect the determination of the spin of the black hole. Broad emission lines are produced only for compact irradiating sources situated close to the black hole. This is the only case where the black hole spin can be unambiguously determined. In all other cases the line shape is narrower, which could either be explained by a low spin or an elongated source. We conclude that for those cases and independent of the quality of the data, no unique solution for the spin exists and therefore only a lower limit of the spin value can be given.

Key words: Accretion, Accretion Discs, black hole physics, Galaxies: Nuclei, galaxies: active, Lines: Profiles

1 INTRODUCTION

Due to the vicinity of the X-ray emitting region in Active Galactic Nuclei (AGN) and X-ray binaries to the central compact object, it is expected that the observed X-ray spectrum will show signs of relativistic effects (Fabian et al. 1989). Such effects were first seen in the skew symmetric shape of the fluorescent Fe $K\alpha$ line from these objects (Tanaka et al. 1995; Reynolds & Nowak 2003, and references therein). These relativistic lines are present in a significant fraction of AGN spectra (Guainazzi, Bianchi & Dovčiak 2006; Nandra et al. 2007; Longinotti et al. 2008; Patrick et al. 2011) and Galactic black hole binaries (Miller 2007; Duro et al. 2011; Fabian et al. 2012a). Recent work has been applying a more self-consistent approach by including models for the relativistic distortion of the full reflection spectrum during the data analysis (see, e.g., Zoghbi et al. 2010; Duro et al. 2011; Fabian et al. 2012b; Dauser et al. 2012).

The relativistic distortion of the reflection spectrum is determined by the spin of the black hole, a , the geometry of the reflec-

tor, and our viewing direction on the system, parametrised through an appropriate inclination angle, i . Additionally a primary source of radiation has to exist in order to produce the observed reflection and also the underlying continuum. Initial assumptions were that the primary source consists of a hot corona around the inner regions of the disc, as Comptonization of soft disc photons in such a corona naturally produces a power law spectrum which fits the observations (Haardt 1993; Dove et al. 1997). Under the assumption that the intensity of the hard radiation scattered back onto the disc by the corona is proportional to the local disc emissivity, the irradiation of the accretion disc would be $I(r) \propto r^{-3}$ for the outer parts, and gradually flatten towards the inner edge of the disc for a standard Shakura & Sunyaev (1973) disc.

With the advent of high signal to noise data from satellites such as *XMM-Newton*, however, measurements showed a disagreement with the Fe $K\alpha$ line profiles predicted by this coronal geometry. For many sources, the data favoured disc emissivities that are much steeper in the inner parts of the accretion disc (see, e.g., Wilms et al. 2001; Miller et al. 2002; Fabian et al. 2002; Fabian et al. 2004, 2012a; Brenneman & Reynolds 2006; Ponti et al. 2010; Brenneman et al. 2011; Gallo et al. 2011;

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Dauser et al. 2012). Variability studies of the broad iron lines pose additional problems for standard corona models. In such studies the time variability of the continuum flux, i.e., the primary hard X-ray radiation, is compared to the flux in the lines, which are produced by the reflected radiation. This allows to probe the connection between the primary and the reflected radiation. In a coronal geometry, one expects a positive correlation between the strength of the relativistically distorted reflection spectrum and the primary continuum (Martocchia & Matt 1996). This is in contrast to what is observed: Measurements of MCG–6-30-15 (Fabian & Vaughan 2003; Miniutti et al. 2003) revealed large variations of the direct radiation, while the reflected component remained constant.

As shown by Martocchia, Matt & Karas (2002), Fabian & Vaughan (2003), Miniutti et al. (2003), and Vaughan & Fabian (2004) for the case of MCG–6-30-15, in a geometry in which the illuminating continuum is assumed to be emitted from a source on the rotational axis at height h above the black hole, strong light bending yields properties of the reflected radiation that are consistent with the observations. Figure 1 illustrates this “lamp post” geometry (Matt, Perola & Piro 1991; Martocchia & Matt 1996). In general, data and predicted line shapes show very good agreement (see Wilkins & Fabian 2011; Duro et al. 2011; Dauser et al. 2012). The lamp post model also explains the observed connection between the luminosity and the reflection strength: For a primary source very close to the black hole, most of the photons are focused on the accretion disc, producing a strong reflection component. Therefore less photons are left over to contribute to the continuum component, which is directly emitted towards the observer (Miniutti & Fabian 2004). For an increasing height of the hard X-ray source this effect gets weaker and thus more photons can escape, which strengthens the continuum radiation and, depending on the flux state of the X-ray source, weakens the reflected flux (Miniutti & Fabian 2004; Miniutti 2006).

Based on the earlier work on the lamp post geometry presented above, Fukumura & Kazanas (2007) provided a more detailed treatment of the emissivity for arbitrary spin and anisotropic emission of the primary source. Off-axis sources were first investigated by Ruszkowski (2000). Using similar methods Wilkins & Fabian (2012) presented a ray tracing method working on Graphics Processing Units (GPUs), which can calculate irradiation profiles for almost arbitrary geometries of the primary sources, now also including sources extended along and perpendicular to the rotational axis. In this *paper* we present a complete method to derive irradiation profiles in the lamp post geometry (Sect. 2), including radially extended and accelerating sources (Sect. 3). Using this formalism, we introduce an implementation of the lamp post geometry as a fitting model for relativistic reflection, which can be applied to more realistic expectations of theoretical jet models (Sect. 4.1). In Sect. 4.2 we analyse the shape of the reflection features predicted by the different types of irradiating sources. In particular we concentrate on the implications for spin measurements, the different assumptions for the geometry of the primary source have (Sect. 4.3). These results are quantified by simulating such observations for current instruments (Sect. 4.4). Finally, we summarise the main results of the paper in Sect. 5.

2 THEORY

2.1 Introduction

We calculate the shape of the relativistic line in the lamp-post geometry by following the radiation emitted from the primary source on the axis of symmetry of the accreting system to the accretion disc and from there to the observer. Due to the deep gravitational potential close to the black hole, the photon trajectories are bent and their energies red-shifted. Moreover the relativistic movement of the accretion disc alters the energy flux incident on the disc and the shape of the observed line through the relativistic Doppler effect. Using techniques introduced by Cunningham (1975), the flux seen from a certain element of the accretion disc under a specific inclination can be predicted (e.g., Speith, Riffert & Ruder 1995) and summed up to the complete spectrum of the source. This problem has been extensively studied in the past and there are several models available to predict the relativistic smearing given a certain emissivity of the accretion disc (e.g., Fabian et al. 1989; Laor 1991; Dovčiak, Karas & Yaqoob 2004; Brenneman & Reynolds 2006; Dauser et al. 2010). In these models the intensity emitted from the accretion disc (the so-called “emissivity”) is parametrised as a power law $r^{-\epsilon}$ with index ϵ , where r is the distance to the black hole. The standard behaviour is $\epsilon = 3$, which is proportional to the energy release in a standard Shakura & Sunyaev (1973) disc.

This emissivity can also be calculated directly from an irradiating source, the so-called “primary source”. In the following we will use a source situated on the rotational axis of the black hole for this purpose. By applying the same ray-tracing techniques used to trace photons from the disc to the observer, we can also calculate the proper irradiation of the accretion disc by the primary source. The radial dependency of this irradiation is equal to the reflected radiation, i.e., the emissivity, which was previously modelled by a power law. In this paper we concentrate on the *irradiation* of the accretion disc. In order to be able to compare our results to observational data, we also need to calculate the ray tracing from the accretion disc to the observer, which is done with the RELLINE-code (Dauser et al. 2010).

2.2 Photon Trajectories in the Lamp Post Geometry

In the following we will concentrate on a simplified geometry by assuming a point-like, photon emitting primary source at a height h above the rotational axis of the black hole. This source irradiates a thin, but optically thick accretion disc (Fig. 1). Relativistic photon trajectories in the lamp post geometry were first investigated by Matt, Perola & Piro (1991) and used by Martocchia & Matt (1996) in order to explain the very large equivalent width of the iron $K\alpha$ line in some AGN. A more detailed discussion of effects in this geometry was presented by Martocchia, Karas & Matt (2000), including a discussion of the influence of the black hole’s spin on the overall spectra.

As a good physical explanation of this hard X-ray source on the rotation axis is the base of a jet (Markoff & Nowak 2004), we call this geometry also the “jet base geometry”. This interpretation is highly supported by the earlier work of Ghisellini, Haardt & Matt (2004), where it is shown that all AGN are capable of forming jets. This is achieved by inventing the concept of “aborted” jets for radio-quiet quasars and Seyferts, which are produced when the velocity of the outflowing material is smaller than the escape speed. Therefore such a jet extends

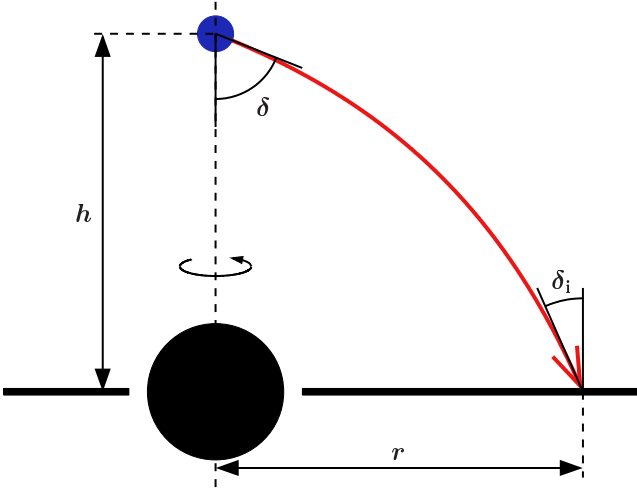


Figure 1. A schematic drawing of the components in the lamp post geometry. The primary source of photons (blue) is situated above the rotational axis of the black hole and is emitting photons (red), which hit the accretion disc.

only a small distance from the black hole, producing only a negligible amount of radio flux while at the same time it strongly irradiates the inner accretion disc in X-rays, which produces the observed, highly relativistic reflection (Ghisellini, Haardt & Matt 2004). This interpretation is encouraged by works showing that direct and reprocessed emission from such a jet base is equally capable in describing the observed X-ray spectrum as a corona above the accretion disc and also yields a self-consistent explanation of the full radio through X-ray spectrum of many compact sources (Markoff, Nowak & Wilms 2005; Maitra et al. 2009). In addition a direct connection between the X-rays and the radio can explain the correlation between observed radio and X-ray flares of Microquasars such as GX 339–4 (Corbel et al. 2000) or Cyg X-1 (Wilms et al. 2007). We note that a similar kind of connection is also indicated in some measurements of AGN like 3C 120 (Marscher et al. 2002) or 3C 111 (Tombesi et al. 2012). Additionally, evidence of a direct influence of the jet on the black hole in Microquasars is growing recently. Namely, Narayan & McClintock (2012) observe a direct correlation between the jet power and the spin value from analysing a small sample of sources.

In the following we will briefly summarise the most important equations required for deriving the photon trajectories from a source on the rotational axis of the black hole. As we are dealing with potentially rapidly rotating black holes, we choose the Kerr (1963) metric in Boyer & Lindquist (1967) coordinates to describe the photon trajectories. Following, e.g., Bardeen, Press & Teukolsky (1972), the general photon momentum is given by

$$p_t = -E, \quad p_r = \pm E\sqrt{V_r}/\Delta, \quad p_\theta = \pm E\sqrt{V_\theta}, \quad p_\phi = E\lambda, \quad (1)$$

with

$$\Delta = r^2 - 2r + a^2 \quad (2)$$

$$V_r = (r^2 + a^2)^2 - \Delta(q^2 + a^2) \quad (3)$$

$$V_\theta = q^2 - \cos^2\theta \left[\frac{\lambda}{\sin^2\theta} - a^2 \right] \quad (4)$$

for a certain distance r and spin a of the black hole. The spin is defined in such a way that its absolute value ranges from $0 \leq |a| \leq 1$, where at the maximal value, $a_{\max} = 1$, the event horizon would

have to rotate at the speed of light.¹ Note that all equations in this paper are given in units of $G \equiv M \equiv c \equiv 1$. The different signs in equation (1) are for increasing (upper sign) and decreasing (lower sign) values of r and θ . Here the conserved quantities are the total energy, E , the angular momentum parallel to the rotational axis of the black hole, λ , and the Carter (1968) constant, q . The latter is given by

$$q = \sin\delta \sqrt{\frac{h^2 - 2h + a^2}{h^2 + a^2}}. \quad (5)$$

Assuming the source to be located on the symmetry axis of the system ($\theta = 0$) simplifies the calculation significantly, as p_θ does only take a real value if $\lambda = 0$, and therefore equation (1) simplifies to

$$p_\mu = E(-1, \pm\sqrt{V_r}/\Delta, \pm E|q|, 0). \quad (6)$$

Then the trajectory can finally be calculated numerically from the integral equation

$$\int_h^r \frac{dr'}{\pm\sqrt{V_{r'}}} = \int_0^\theta \frac{d\theta'}{\pm\sqrt{V_{\theta'}}}. \quad (7)$$

(following Chandrasekhar 1983; Speith, Riffert & Ruder 1995). Note that the sign changes at turning points of the photon trajectory. In this case the left part of equation (7) has to be split into two integrals, each going from and to the turning point, respectively. Note that no turning points in the θ direction have to be taken into account, as photons which initially fly towards the disc, will not exhibit a turning point before crossing the equatorial plane and thus before hitting the accretion disc.

2.3 Illumination of the Accretion Disc

In order to calculate the incident intensity on the accretion disc, we first have to consider the geometric effects intrinsic to the lamp post setup. Without any relativistic effects the intensity impinging on the accretion disc for an isotropic primary emitter is given by

$$I_i(r, h) \propto \frac{\cos\delta_i}{r^2 + h^2} = \frac{h}{(r^2 + h^2)^{\frac{3}{2}}}. \quad (8)$$

This means that already for flat space time the irradiated intensity strongly depends on the radius.

Due to the strong gravity the photon trajectories will be significantly bent, i.e., in our setup the photons will be “focused” onto the inner regions of the accretion disc (Fig. 1), modifying the radial intensity profile. Note that this focusing depends on both, the height, h , and the initial direction of the photon, parametrised by the angle between the system’s axis of symmetry and the initial direction of the photon, δ .

Using the equations of the previous section we developed a ray-tracing code using similar techniques as those presented in Dauser et al. (2010). With this code we are able to calculate photon trajectories from the point of emission (h, δ) at the primary source to the accretion disc, yielding the location (r, δ_i) where this specific photon hits the disc. As the primary source is located on the rotational axis of the black hole, the trajectory of the photon is uniquely determined by the q -parameter (equation 5). The incident point is then calculated by solving the integral equation (7) for r in the case of $\theta = \pi/2$.

¹ Thorne (1974) showed that the realistic upper limit of the spin is more likely $a_{\max} = 0.998$.

Knowing where the isotropically emitted photons hit the accretion disc, we can derive the photon flux incident on its surface. As the photons are designed to be emitted at equally spaced angles δ , the distance Δr between these points is related to the incident intensity. Photons emitted in $[\delta, \delta + \Delta\delta]$ are distributed on a ring on the accretion disc with an area of $A(r, \Delta r)$. The proper area of such a ring at radius r with thickness Δr is given by

$$A(r, \Delta r) = 2\pi r \cdot \sqrt{\frac{r^4 + a^2 r^2 + 2a^2 r}{r^2 - 2r + a^2}} \Delta r \quad (9)$$

in the observer's frame of rest (Wilkins & Fabian 2012). In order to calculate the irradiation in the rest frame of the accretion disc, we have to take into account its rotation at relativistic speed. The area of the ring will therefore be contracted. Using the Keplerian velocity profile deduced from the Kerr metric (Bardeen, Press & Teukolsky 1972), the disc's Lorentz factor is

$$\gamma^{(\phi)} = \frac{\sqrt{r^2 - 2r + a^2}(r^{3/2} + a)}{r^{1/4} \sqrt{r\sqrt{r} + 2a - 3\sqrt{r}\sqrt{r^3 + a^2 r + 2a^2}}} \quad (10)$$

(Bardeen, Press & Teukolsky 1972, see also Wilkins & Fabian 2011, 2012). Taking into account that the photons are emitted at equally spaced angles, we finally find that for isotropic emission the geometric contribution to the incident intensity has to be

$$I_i^{\text{geo}} = \frac{\sin \delta}{A(r, \Delta r) \gamma^{(\phi)}} \quad (11)$$

Because of the relative motion of the emitter and the accretion disc, as well as because of general relativistic effects, the irradiated spectrum will be shifted in energy (Fukumura & Kazanas 2007). Using the initial four-momentum at the primary source

$$u_h^\mu = (u_h^t, 0, 0, 0) \quad (12)$$

and the corresponding four-momentum on the accretion disc

$$u_d^\mu = u_d^t(1, 0, 0, \Omega) \quad (13)$$

together with the photon's momentum (equation 6), the energy shift is

$$g_{\text{lp}} = \frac{E_i}{E_e} = \frac{p_\mu u_d^\mu}{p_\nu u_h^\nu} = \frac{(r\sqrt{r} + a) \sqrt{h^2 - 2h + a^2}}{\sqrt{r}\sqrt{r^2 - 3r + 2a\sqrt{r}\sqrt{h^2 + a^2}}} \quad (14)$$

The components of the four-velocities are calculated from the normalising condition $u_\mu u^\mu = -1$ (see, e.g., Bardeen, Press & Teukolsky 1972).

As the number of photons is conserved we can write

$$N_e^{(\text{ph})} \Delta t_e \Delta E_e = \text{const.} = N_i^{(\text{ph})} \Delta t_i \Delta E_i \quad (15)$$

where $N_e^{(\text{ph})}$ ($N_i^{(\text{ph})}$) is the emitted (incident) photon flux. Assuming a power law shape of the emitted radiation

$$N_e^{(\text{ph})} = E_e^{-\Gamma} \quad (16)$$

the photon flux on the accretion disc is given by

$$N_i^{(\text{ph})}(r, a) = E_i^{-\Gamma} \cdot g_{\text{lp}}(r, a)^\Gamma \quad (17)$$

as $\Delta E_e / \Delta E_i = 1/g_{\text{lp}}$ and $\Delta t_e / \Delta t_i = g_{\text{lp}}$. Due to the relativistic energy shift, the incident photon flux now also depends on where the photon hits the accretion disc (r) and which spin the black hole has. Using this result, we can finally calculate the incident flux on the accretion disc

$$F_i(r, h) = I_i^{\text{geo}} \cdot g_{\text{lp}}^\Gamma = \frac{\sin \delta g_{\text{lp}}^\Gamma}{A(r, \Delta r) \gamma^{(\phi)}} \quad (18)$$

This is in line with the results obtained by Fukumura & Kazanas (2007)².

In order to understand the influence of the different relativistic parameters on the incident intensity, Fig. 2 shows the single components of equation (18). A similar discussion of these components is also given by Wilkins & Fabian (2012). We assume that the primary source is an isotropic emitter. All effects are strongest for small radii and will therefore be most important for high spin, where the accretion disc extends to very low radii. First, length contraction reduces the area of the ring as seen from the primary source. In the rest frame of the accretion disc, this ‘‘contraction’’ implies an effectively larger area and therefore the incident flux decreases with increasing v^ϕ proportional to the inverse Lorentz factor $1/\gamma^{(\phi)}$ (Fig. 2a). When compared to flat space time, the area of disc close to the black hole is additionally enhanced in the Kerr metric (Fig. 2b). Interestingly, this effect is almost independent of the spin of the black hole³. However, compared to the effect induced by the energy shift (equation 14), the change in area is only a minor effect. Depending on the power law index, Γ , the energy shift of the photons hitting the disc is the strongest factor influencing the reflection spectrum. For a source on the rotational axis of the black hole the change in irradiated flux can be as large as a factor of 100 (Fig. 2c), depending strongly on the steepness of the primary spectrum. This amplification factor depends on the height of the emitting source (it becomes larger for increasing height) and decreases for larger radii. Especially for low h light bending focuses photons towards the disc and additionally enhances the irradiation of the inner regions. The dashed lines in Fig. 2d show how $1/\Delta r$ decreases in flat space just due to geometrical reasons following equation (8). The fully relativistic treatment (solid lines) reveals a focusing of the photons towards the black hole. But compared to the ‘‘effectively enhanced’’ irradiation of the inner parts due to the energy shift (Fig. 2c), the relativistic focusing is only a minor effect. In summary, the power law index, Γ , has the strongest influence on the irradiation profile at small radii, while the height of the emitting source mostly affects the outer parts of the disc.

Finally, Fig. 2e combines all effects and shows the incident flux in the rest frame of the accretion disc. In general this plot confirms our overall understanding of the lamp post geometry: Sources at low height strongly irradiate the inner parts but almost not the outer parts. For an increasing height of the source more and more photons hit the outer parts of the accretion disc and an increasing region of more constant irradiation at roughly $h/2$ is created. In order to check our simulation for consistency, a thorough check against the calculations of Fukumura & Kazanas (2007) was done and we could validate the result from Fig. 2e at high precision. The same is true for the stationary point source solution of Wilkins & Fabian (2012).

2.4 Emissivity Profiles in the Lamp Post Geometry

Since for a simple accretion disc the local disc emissivity is roughly $\propto r^{-3}$, in the description of observations it is common to parametrise the disc emissivity profile through

$$F(r, h) \propto r^{-\epsilon} \quad (19)$$

² Note that Fukumura & Kazanas (2007) use the spectral index α , whereas we use the photon index Γ . Both quantities are related by $\Gamma = \alpha + 1$.

³ deviations are less than 0.2%

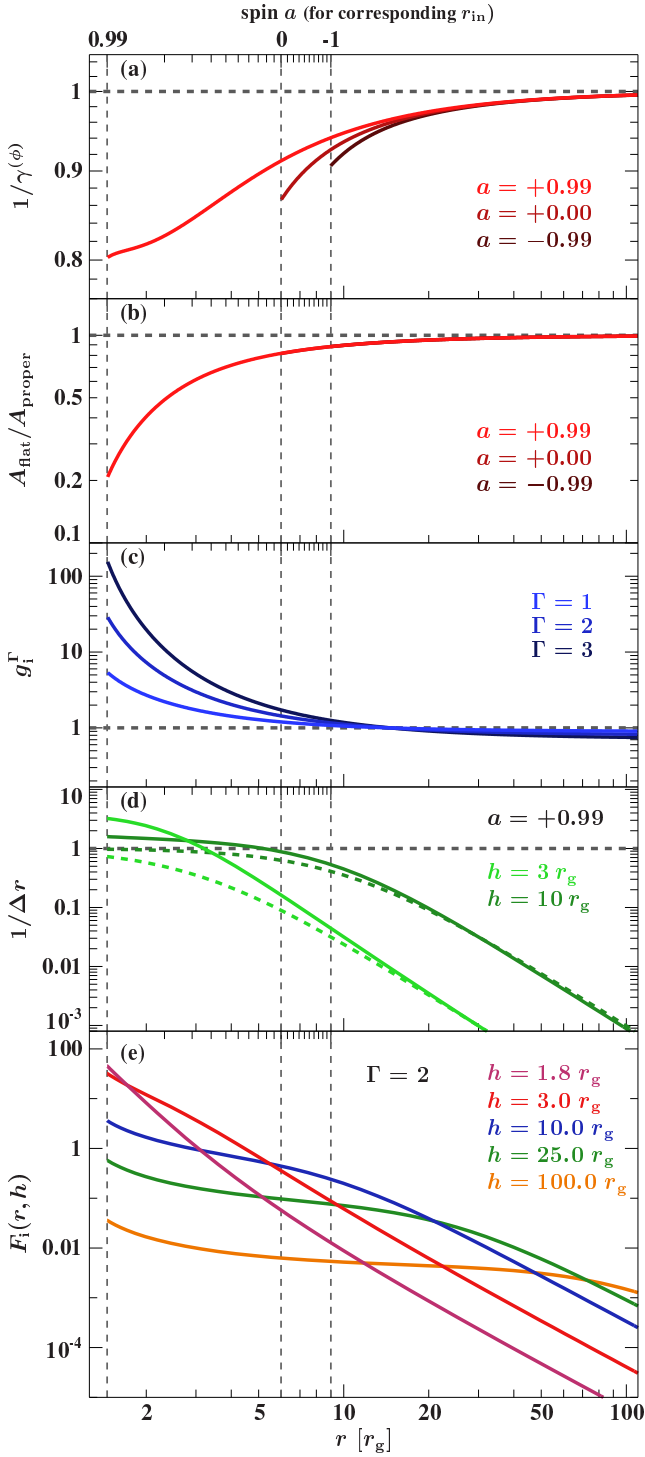


Figure 2. Relativistic factors which influence the incident flux on the accretion disc compared to the emitted intensity at the primary source (see equation 18). If not stated in the figure explicitly we use $a = 0.99$ and assume that the primary source is an isotropic emitter. The vertical dashed lines indicate the location of the innermost stable circular orbit (ISCO) for certain values of spin. (a) The inverse beaming factor (red), which determines the influence of length contraction (see equation 10) on the incident flux. (b) The impact of the proper area. (c) Geometric intensity distribution on the accretion disc for the relativistic (solid) and Newtonian case (dashed). (d) Energy shift, which the photon experiences when travelling from the primary source to the accretion disc, taken to the power of Γ (blue). (e) Combined irradiating flux on the accretion disc for a primary source at different heights but equal luminosity.

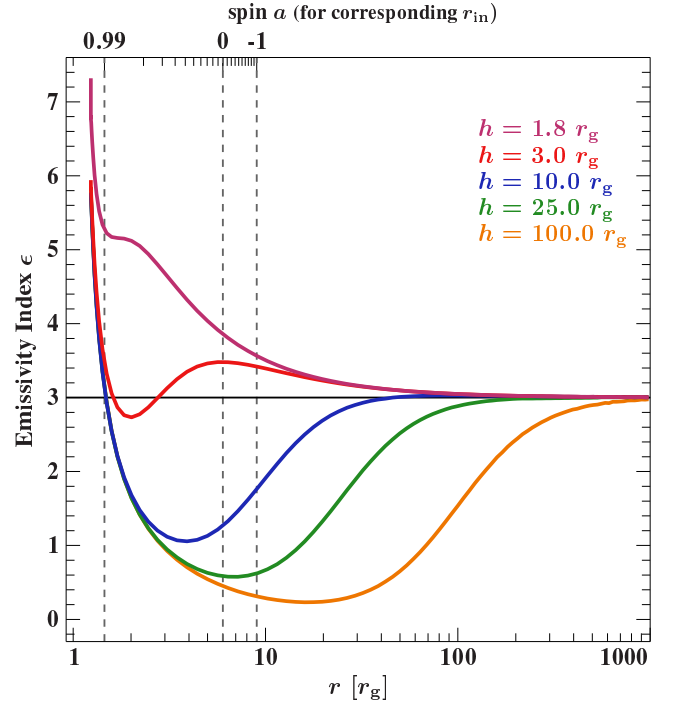


Figure 3. The emissivity index ϵ as defined in equation (19) of the radiation irradiating the accretion disc from a primary source at different heights h .

where ϵ is called the emissivity index. Note that in this representation the information of the normalisation of the emissivity profile is lost. But as usually the luminosity of the irradiating source is not known, this is not very important for reflection studies. Our calculations easily allow to determine the radius dependent emissivity index and thus phrase our results in a language that is directly comparable with observations.

Figure 3 shows the emissivity profile for different heights of the primary source. Regardless of the specific height, three different radial zones are visible in all profiles. Firstly, for large radii the index converges always towards its value in flat space ($\epsilon = 3$, see equation 8). The closer the emitting source is to the black hole, the faster ϵ converges towards this value.

The zone closer than $2 r_g$ from the black hole is characterised by a strong steepening of the emissivity profile towards the inner edge of the accretion disc. Except for an extremely low height, this steepening is almost independent of the height of the primary source⁴. Hence, a large emissivity at low radii, which is usually interpreted as “strong focusing of a low height emitter”, is not directly related to the height of the primary source. Instead, the steepness almost solely depends on the relativistic boosting of the primary photons and especially on the steepness Γ of the primary spectrum (see Fig. 2c).

As has been mentioned in the introduction (Sect. 1), many sources are observed to have very steep emissivity indices, i.e., values of $\epsilon = 5-10$ are normal. Similar to the emissivity profile in Fig. 3, we can also derive the maximal possible emissivity index for a certain value of spin and steepness of the input spectrum. For a standard lamp post source at a height of at least $3 r_g$, this infor-

⁴ Clearly, the absolute flux for a certain luminosity is highest for a low emitter (see Fig. 2e), but we usually do not measure the absolute intensity, but only the emissivity index.

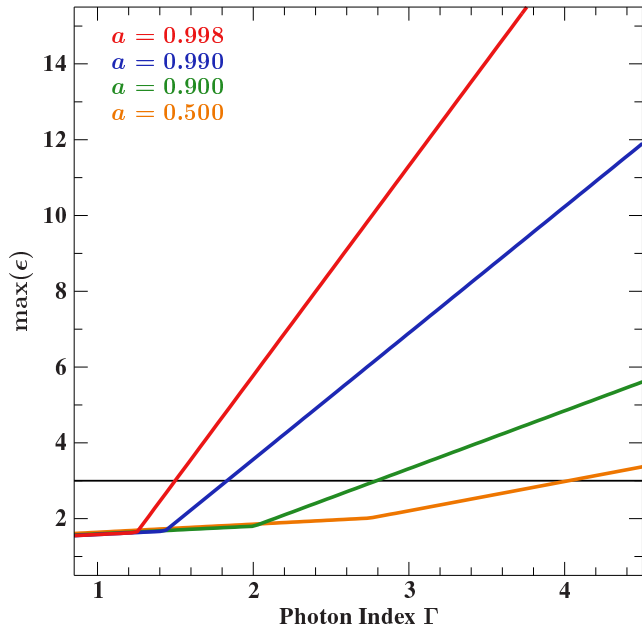


Figure 4. The maximum possible emissivity index at the inner regions of the disc ($r < 10 r_g$) for a certain photon index Γ . The height of the source was chosen to be $h = 10 r_g$. However, note that we showed in Fig. 3 that for $h > 3 r_g$ the steepening at the inner edge of the disc is almost independent of the height.

mation is plotted in Fig. 4. In this case, no large emissivities are expected if the black hole is not a maximal rotator, i.e., roughly for $a < 0.9$. If the input spectrum is hard ($\Gamma < 2.5$), the emissivity index is flatter than the standard index of $\epsilon = 3$. Even for highly rotating black holes the actual emissivity at the inner edge of the accretion disc is only steep if the incident spectrum is very soft ($\Gamma > 2.5$).

If we want to apply the information of Fig. 4 to a certain observation, we have to take into account that the emissivity is generally parametrised in form of a broken power law. In detail this means that below a certain “break radius” (r_{br}) one single emissivity index is used to describe the steep emissivity and above that it is usually fixed to the canonical r^{-3} behaviour. Generally, break radii are found to be in the range of $3 r_g$ (e.g., in 1H0707–495, Dauser et al. 2012) up to $6 r_g$ (e.g., in NGC 3783, Brenneman et al. 2011).⁵ Hence, the emissivity index we measure in observations accounts for the average steepness in the range of r_{in} to r_{br} and will therefore be lower than the maximal emissivity. We note that the emissivity indices found in many observations are close to or above the maximal allowed emissivity index as defined in Fig. 4. For example $\epsilon \approx 5$ for $\Gamma = 1.8$ in NGC 3783 (Brenneman et al. 2011), $\epsilon \approx 6$ for $\Gamma = 2$ in MCG–6-30-13 (Brenneman & Reynolds 2006), $\epsilon > 6.8$ for $\Gamma = 1.37$ in Cygnus X-1 (Fabian et al. 2012a), or $\epsilon \approx 10$ for $\Gamma = 3.3$ in 1H0707–495 (Dauser et al. 2012). As the emissivity index obtained from these measurements is averaged over the innermost few r_g these emissivities can therefore not be properly explained solely by the lamp post geometry. Despite these issues, however, in some cases it is possible to use Fig. 4 to decide if a measured value for the emissivity index can reasonably be explained in the lamp post geometry. This method has been successfully applied

⁵ Note that the value of the break radius is highly correlated with the emissivity index when trying to constrain both by observation.

by Duro et al. (2011) to find a unique and consistent solution to describe the reflection spectrum of Cyg X-1.

2.5 The incident angle

The incident angle δ_i of the irradiated radiation is important for modelling the reflected spectrum, as it determines the typical interaction depth of the reflected photon and therefore strongly influences the limb-darkening of the reflected radiation (e.g., Svoboda et al. 2009). Constructing a normal vector on the disc, δ_i is given by

$$\cos \delta_i = \frac{p_{\perp}}{|p|} = \frac{(p_d)_{\mu} \left(n_d^{(\theta)} \right)^{\mu}}{(p_d)_{\nu} (u_d)^{\nu}} \Bigg|_{\theta=\pi/2} = \frac{q}{r u_d^t(r, a)}. \quad (20)$$

Figures 5a and b show δ_i for different heights of the primary source and assuming an isotropic primary emitter. For most disc radii the photons hit the disc at a shallow angle, except for a small fraction of disc. The location and width of the steeper in-falling photons depends on the height of the primary source.

The effect of the incidence angle of the illumination in the reflected spectrum from an accretion disc has been discussed in García & Kallman (2010). In the calculation of reflection models, the boundary at the surface of the disc is defined by specifying the intensity of the radiation field that illuminates the atmosphere at a particular angle. Using their equation (19) and (37), this can be expressed as

$$I_{inc} = \left(\frac{2n}{4\pi} \right) \frac{\xi}{\cos \delta_i}, \quad (21)$$

where n is the gas density (usually held fixed), and ξ is the ionisation parameter that characterises a particular reflection model (Tarter, Tucker & Salpeter 1969). Consequently, for a given ionisation parameter, varying the incidence angle varies the intensity of the radiation incident at the surface. This has interesting effects on the ionisation balance calculations. If the photons reach the disc at a normal angle ($\delta_i = 0$), the intensity has its minimum value, but the radiation can penetrate into deeper regions of the atmosphere producing more heating. On the contrary, for grazing incidence ($\delta_i = 90$) I_{inc} increases, resulting in a hotter atmosphere near the surface; but the radiation field thermalizes at smaller optical depths, which yields lower temperature in the deeper regions of the disc. Evidently, these changes in the ionisation structure will also affect the reflected spectrum (see Fig. 5c). The narrow component of the emission lines are expected to be emitted relatively near the surface, where photons can easily escape without being absorbed or scattered. On the other hand, the broad component of the emission lines, and in particular the ones from high Z elements such as iron, are produced at larger optical depths ($\tau \sim 1$), and therefore are more likely to be affected by changes in the ionisation structure of the slab.

However, García & Kallman (2010) showed that in a general sense, reflected spectra resulting from models with large incidence angles tend to resemble models with higher illumination. This means that the changes introduced by the incidence angle can be mimicked by correcting the ionisation parameter to account for the difference introduced in the illumination. The current analysis shows that below $7 r_g$ the incidence angle can vary as much as 25 – 80 degrees, equivalent to a change in the ionisation parameter by more than a factor of 5. Figure 5c shows the reflected spectra for these two incidence angles predicted by the XILLVER code (García & Kallman 2010;

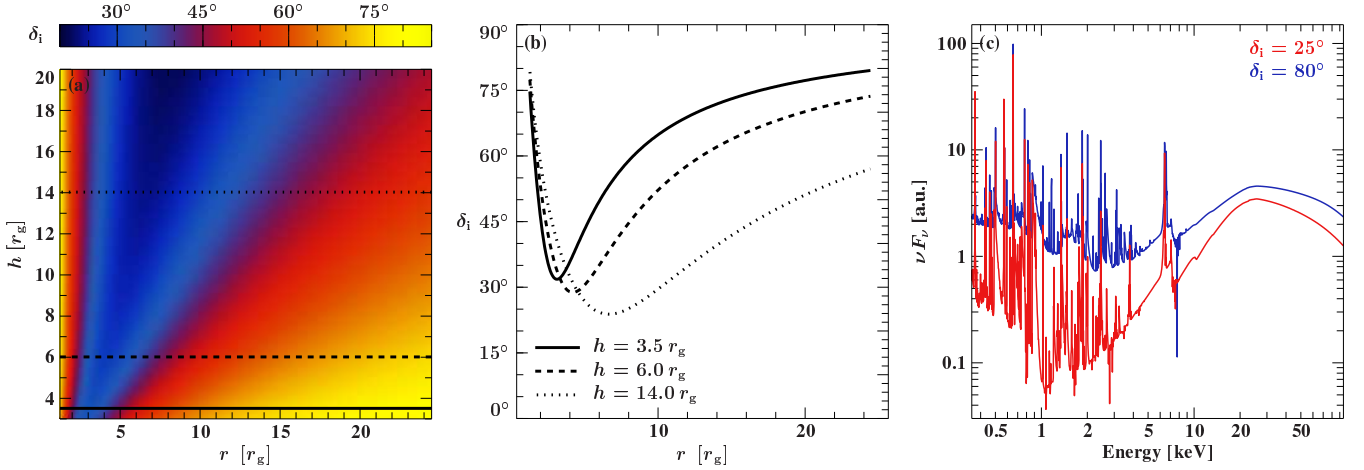


Figure 5. (a) 2d image showing the incident angle δ_i of photons on the accretion disc. The spin of the black hole is $a = 0.998$. (b) incident angle for the three lines (solid, dashed, dotted) marked in subfigure (a). (c) Sample reflection spectra for different incident angles $\delta_i = 25^\circ$ (red) and 80° (blue) calculated with the XILLVER code (García & Kallman 2010; García, Kallman & Mushotzky 2011).

García, Kallman & Mushotzky 2011) using the same ionisation parameter. The effect of the incidence angle is evident.

3 AN EXTENDED RAY-TRACING CODE

So far we assumed that the emitting primary source is at rest with respect to the black hole. If the primary emitter is the base of the jet, however, then it is far more likely that the primary source is moving. Typical speeds at the jet base can already be relativistic (McKinney 2006). Furthermore, if the irradiating source is a jet, then we need to relax our assumption of a point-like emitter and include the radial extent of the jet. As we show in this section by taking into account both of these extensions, the line shape is significantly affected.

3.1 A Moving Jet Base

First investigations of a moving source irradiating the accretion disc and its implication for the Fe $K\alpha$ were done by Reynolds & Fabian (1997). Beloborodov (1999) investigated the coupling between the moving primary source and the reflected radiation. Using general relativistic ray tracing techniques, Fukumura & Kazanas (2007) and Wilkins & Fabian (2012) calculated the illumination profile on the accretion disc.

We first assume that the emitting primary point source is moving at a constant velocity $\beta = v/c$. The most prominent effect of a moving jet base compared to a jet base at rest is the Doppler boosting of radiation in the direction of the moving blob, i.e., away from the accretion disc. This boosting also means that the energy shift of the photon between primary source and accretion disc changes with velocity. The 4-velocity is then given by

$$u_h^\mu = u_h^t \left(1, \frac{dr}{dt}, 0, 0 \right) \quad (22)$$

The velocity β as seen by an observer at the same location in the locally non-rotating frame (LNRF) is connected to dr/dt through

$$\beta = \frac{e_\mu^{(r)} u_h^\mu}{e_\nu^{(t)} u_h^\nu} = \frac{r^2 + a^2}{\Delta} \cdot \frac{dr}{dt} \quad (23)$$

where $e_\mu^{(\nu)}$ are the tetrad basis vectors for $\nu = t, r, \theta, \phi$ (see Bardeen, Press & Teukolsky 1972).

In order to calculate the trajectory of a photon emitted at an angle δ' from the moving source, we transform from the moving frame to the stationary, locally non-rotating frame at the same location. This means that the photon is emitted at an angle δ in the stationary system according to

$$\cos \delta = \frac{\cos \delta' - \beta}{1 - \beta \cos \delta'} \quad (24)$$

depending on the velocity of the source. Following, e.g., Krolik (1999), it can be easily shown that this approach implies that the intensity observed in the stationary frame will be altered by a factor of

$$\mathcal{D}^2 \quad (25)$$

where \mathcal{D} is the special relativistic Doppler factor, which is defined in our case as

$$\mathcal{D} = \frac{1}{\gamma(1 + \beta \cos \delta)} \quad (26)$$

with $\gamma = 1/\sqrt{1 - \beta^2}$ being the Lorentz factor of the moving source. Using the transformed intensity, we can now apply the stationary calculations from Sect. 2.3 to the new emission angle δ to obtain the irradiation of the accretion disk by a moving source.

Additionally, we have to calculate the proper energy shift between the accretion disc and the moving source, with the 4-velocity given by equation (22). Similarly to equation (14) this energy shift is given by

$$g_{1p} = \frac{(p_a)_\mu u_a^\mu}{(p_h)_\nu u_h^\nu} = \frac{g_{1p}(\beta = 0)}{\gamma \left(1 \mp \frac{\sqrt{(h^2 + a^2)^2 - \Delta(q^2 + a^2)}}{h^2 + a^2} \beta \right)} \quad (27)$$

Note that for large heights g_{1p} simplifies to its special relativistic limit, $g_{1p} = \mathcal{D}g_{1p}(\beta = 0)$. Using the results we already obtained for a stationary primary source (see Sect. 2.3), we can finally write down the total flux the accretion disc sees from a moving source:

$$F_i(r, h, \beta) = \frac{\mathcal{D}^2 F_i(r, h, \beta = 0)}{\left[\gamma \left(1 \mp \frac{\sqrt{(h^2 + a^2)^2 - \Delta(q^2 + a^2)}}{h^2 + a^2} \beta \right) \right]^\Gamma} \quad (28)$$

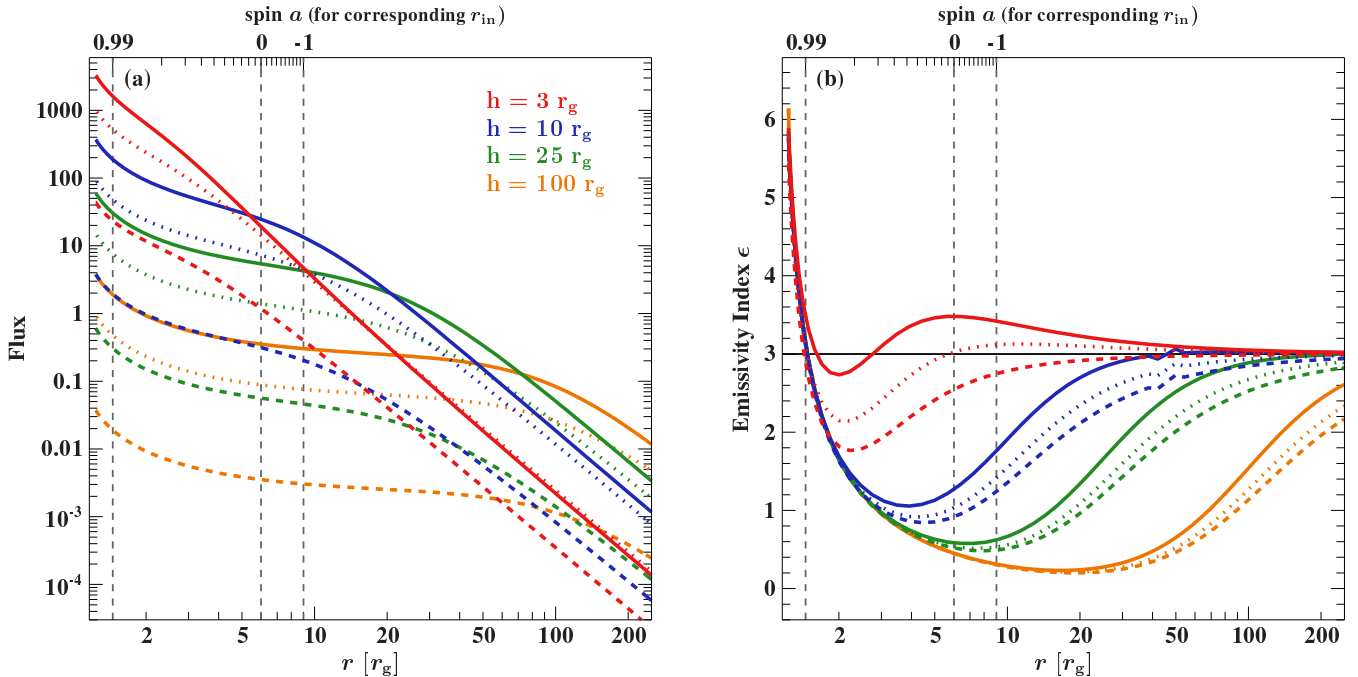


Figure 6. (a) Irradiating flux and (b) Emissivity profiles similar to Fig. 3, which show the impact of a moving jet base with velocities $v = 0 c$ (solid), $v = 0.5 c$ (dotted), and $v = 0.9 c$ (dashed).

Figure 6 shows the dependency of the emissivity index on the velocity of the jet base. In general the irradiating flux decreases significantly with increasing speed of the jet base, as the photons are boosted away from the black hole (Fig. 6a).

Comparing the shape of the emissivity profiles for a moving jet base (Fig. 6b), the intermediate region of the accretion disc ($3\text{--}100 r_g$) experiences an increase of irradiation with increasing velocity of the jet base. On the other hand the emissivity profile of the very inner regions of the accretion disc does not depend on the movement of the jet. For high spin ($a > 0.9$) the accretion disc extends down to these very small radii and due to the steep emissivity most of the reflected radiation comes from there. If the accretion disc only extends down to $6 r_g$, as is the case for $a = 0$, the irradiation of the innermost regions can differ almost up to a factor of 2 in emissivity index depending on the velocity of the jet base.

3.2 Irradiation by an Elongated Jet

It is straightforward to extend the previous discussion of a moving jet base to the case of an extended jet (see also Wilkins & Fabian 2012). We simply describe the incident radiation for an elongated jet by many emitting points at different heights, weighted by the distance between these points. Emissivity profiles for the extended jet are shown in Fig. 7. In general, the shape of the emissivity profile in the case of an extended source does not differ significantly from that of a point-like source. Similar to the moving jet base (see Sect. 3.1), the irradiation of the inner regions of the accretion disc ($r < 2 r_g$) only differs in normalisation but not in shape (Fig. 7). However, the regions of the disc that are a little bit further outwards ($> 3 r_g$) are affected at a much greater fraction by extending the emission region. But despite these large differences, the overall shape of the emissivity profile does not change, i.e., that the general properties analysed in Sect. 2.4 are still valid in the case of elongated jets.

The influence of changing the location of the jet base while fixing the top height is depicted in Fig. 7a,b. Interestingly the emissivity profile is not very sensitive to the location of the jet base. On the other hand, fixing the jet base at a low value ($3 r_g$) and then increasing the radial extent of the jet (Fig. 7c) strongly alters the emissivity profile. The more the top is away from the base, the larger the deviations become compared to the profile of the jet base (red, dashed line) and the more the irradiation resembles the one from the upper part of the jet. Comparing the extended profiles to the ones for a point-like primary source (dashed lines) reveals that the extended emission creates an irradiation pattern that could have similarly been produced by a point source at an intermediate height in between h_{base} and h_{top} , too. This implies that if we measure an emissivity profile similar to one for a low source height (Fig. 7, dashed red line), the jet cannot be extended or there is no significant amount of radiation from the upper parts irradiating the disc. One way to explain the lack of photons is that the jet does not have a uniform velocity, but the jet base is at rest and from there on the particles are very efficiently accelerated (see McKinney 2006) such that the radiation is beamed away from the accretion disc.

3.3 Jet with Constant Acceleration

Having analysed the effect of a moving primary source and the profile of an extended jet, we are able to combine these effects to form a more realistic approach. It is likely that the actual base of the jet is stationary or has at least a velocity normal to the disc plane that is much less than the speed of light. Above the jet base the particles are efficiently accelerated (McKinney 2006) to higher energies and in the end to very fast velocities seen, e.g., in Very Long Baseline Interferometry (VLBI) measurements (see, e.g., Cohen et al. 2007). In the following we will assume the simplest case by using a constant acceleration \mathcal{A} of the particles. In this case the velocity evolves as (see, e.g., Torres Del Castillo & Pérez Sánchez 2006)

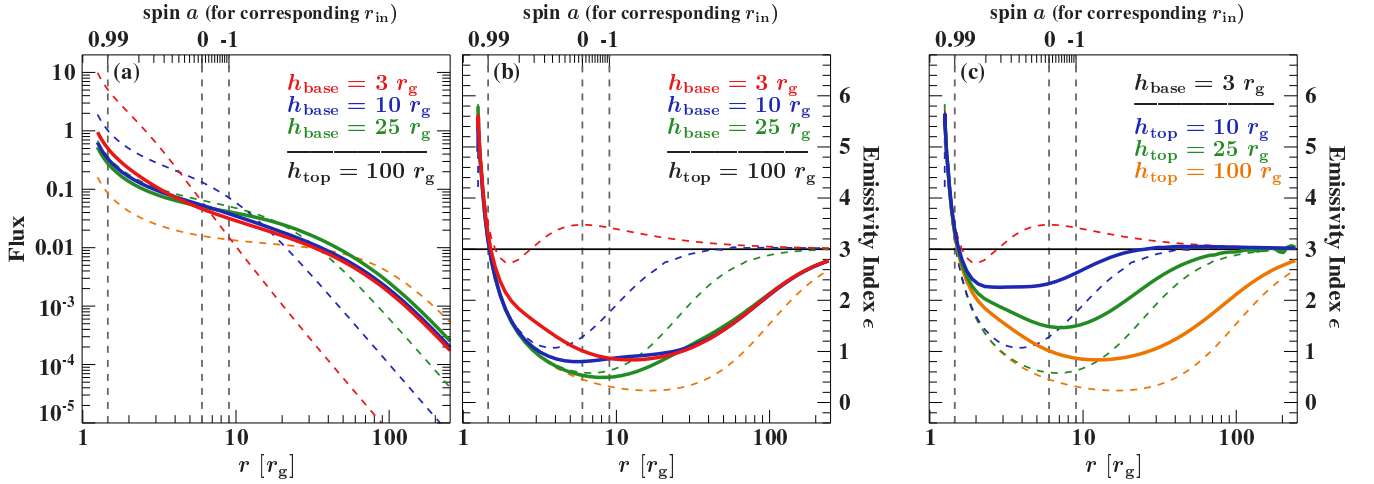


Figure 7. Emissivity profiles similar to Fig. 3, which show the impact of an elongated jet compared to a point source. For comparison the dashed lines show the emissivity of a point-like emitting source with same colours as in Fig. 3, i.e., red, blue, green, and orange for $3 r_g$, $10 r_g$, $25 r_g$, and $100 r_g$, respectively. For varying the base of the emitting region the irradiating flux (a) and the emissivity profile (b) is shown. In (c) the top height of the jet is altered.

$$\beta(t) = \frac{\mathcal{A}t}{\sqrt{c^2 + \mathcal{A}^2 t^2}}, \quad (29)$$

where the time t is given by

$$t = \sqrt{\frac{x^2}{c^2} + 2\frac{x}{\mathcal{A}}} \quad (30)$$

for $x = h - h_{\text{base}}$. Therefore the acceleration to reach a specific velocity β at height h is given by

$$\mathcal{A} = \frac{\gamma - 1}{h - h_{\text{base}}}, \quad (31)$$

where γ is the Lorentz factor. Figure 8a displays the velocity (equation 29) inside the jet for constant acceleration.

The irradiation of the accretion disc in this setup is shown in Fig. 8b. The effect of the accelerated movement shows up at larger radii by steepening the emissivity compared to the stationary jet and the for all heights the profile gets more similar to a point source at the jet base. This result confirms the general picture that for a large acceleration the accretion disc sees only the lowest part, as most of the upper part of the jet is strongly beamed away from the disc. This means that if we measure a localised, low height of the emitting source, it could also be the base of a strongly accelerating jet. Additionally, Fig. 8c reveals that the stronger the acceleration, the more the emissivity profiles resembles the canonical r^{-3} case for all but the innermost radii ($r > 2 r_g$).

4 DISCUSSION

4.1 RELLINE_LP — A new relativistic line model

Using the approach mentioned in Sect. 2.3, the lamp post geometry was incorporated in the RELLINE model (Dauser et al. 2010). This model was designed to be used with common data analysis tools such as XSPEC (Arnaud 1996) or ISIS (Houck & Denicola 2000) for modelling relativistic reflection. It can either predict a single line shape or it can be used as a convolution model smearing a complete ionised spectrum (such as the REFLIONX model Ross & Fabian 2007). The new RELLINE_LP model can be downloaded from <http://www.sternwarte.uni-erlangen.de/research/relativistic>

The calculations in the lamp post geometry are used to determine the proper irradiation profile and replace the artificial broken power law emissivity in the RELLINE model. Besides the standard point source in the lamp post geometry, we also included the extended geometries presented in Sect. 3, i.e., elongated and moving primary sources. As the information of the ray tracing is tabulated, the RELLINE_LP model is evaluated very quickly and thus well suited for data modelling.

4.2 Influence of the lamp post parameters on the shape of the reflection features

With the RELLINE_LP model it is possible to calculate the predicted line shapes of broad emission lines for the different parameters determining the setup in the lamp post geometry. Figure 9 shows that the line shape is very sensitive for certain parameter combinations and almost independent in other cases.

The line shape is highly sensitive to a change in height of the primary source (Fig. 9a and b). Especially when assuming a rapidly rotating black hole, the line shape dramatically changes from a really broad and redshifted line to a narrow and double-peaked structure when increasing the height of the source (Fig. 9a). The same behaviour can be observed for a negatively rotating black hole (Fig. 9b), but here differences are not as large as in the previous case. Broad lines seen from a configuration of a low primary source and a highly rotating black hole are also sensitive to the photon index Γ of the incident spectrum (Fig. 9c). In this case the line shape gets broader for a softer incident spectrum. When fixing the height of the irradiating source (Fig. 9d–f), for a low source height the shape is still sensitive to the spin (Fig. 9d). But already for a medium height of $25 r_g$ the line shapes virtually coincide for all possible values of black hole spin. In this case, even for a rapidly rotating black hole, the line shape does not depend on the steepness of incident spectrum (Fig. 9f).

Using the RELLINE_LP-code, we are also able to compare line shapes for moving (Sect. 3.1), elongated (Sect. 3.2), and accelerating (Sect. 3.3) primary sources. A change in velocity only alters the line shape if the source is at low height and the black hole is rapidly rotating (Fig. 9g). Changing either of these to negatively rotating (Fig. 9h) or a larger source height (Fig. 9i), different ve-

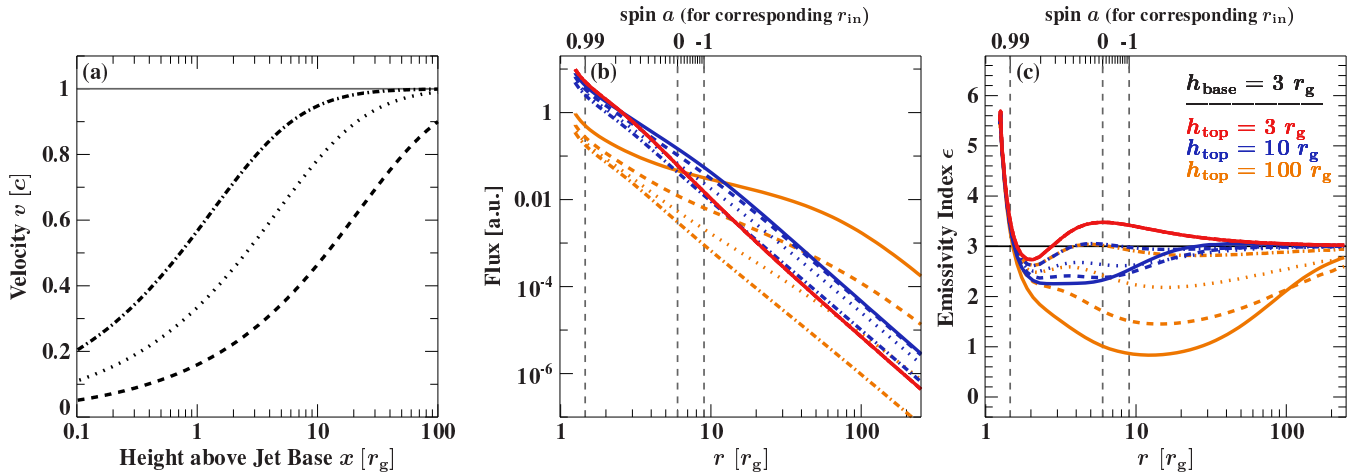


Figure 8. (a) Velocity of the jet assuming a constant acceleration and the jet base at rest. The spin of the black hole is $a = 0.998$, and the power law index $\Gamma = 2$. The acceleration is parametrised by specifying the velocity v_{100} the jet has at $100 r_g$ above the jet base. Lines are plotted for $v_{100} = 0.9$ c (dashed), $v_{100} = 0.99$ c (dotted), and $v_{100} = 0.999$ c (dashed-dotted). (b) and (c) Emissivity profiles for an extended jet with the constant acceleration shown in (a). Parameters are the same as in (a). The solid line displays the emissivity profile for a stationary jet. Despite the different heights, each jet is assumed to have an equal luminosity.

locities of the irradiating source only have a marginal effect on the line shape. Similarly, measuring the radial extent of the primary source is also not possible for all parameter combinations. Firstly, the restrictions of measuring a and Γ for larger heights, as seen in Fig. 9e and f, also apply here. Therefore we fix these two values at $a = 0.99$ and $\Gamma = 2.0$. Setting the base of the source at $3 r_g$ and altering its height (Fig. 9j) does indeed result in great changes in the line shape. These changes are very similar to a change in height of a point-like primary source (see Fig. 9a). Hence, an elongated jet produces a reflection feature similar to a point-like source with an effective height. A similar behaviour can be observed when changing the base of the primary source and leave the upper boundary constant at $100 r_g$. However, as the large upper part now dominates the irradiation of the accretion disc, the profile is not very sensitive to the location of the base of the source. Finally, an accelerating jet (Fig. 9l) only influences the line shape if already relatively high velocities are obtained at a height of $100 r_g$ ($v_{100} > 0.99c$), as in the upper part of the jet more and more photons are beamed away from the accretion disc due to the highly relativistic movement of the emitting medium. The line profiles for a strong acceleration thus resemble very closely the ones for a lower top of the emitting source (Fig. 9j), which means that in reality we will not be able to measure if the emitting source is accelerating without knowing the full geometry of such a primary source.

In some sources with a low mass accretion rate, the disc might be truncated further away from the black hole than the ISCO (see Esin, McClintock & Narayan 1997 and, e.g., the observations by Markowitz & Reeves 2009; Svoboda, Guainazzi & Karas 2010). Generally, relativistic emission lines from such truncated discs are even narrower than for non or negatively spinning black holes. However, such lines can also be explained by the irradiation from an largely elevated (Chiang et al. 2012) or elongated source (Fig. 9k).

Besides the spin, the inclination of the system has also a strong effect on the line shape (see, e.g., Dauser et al. 2010). As the inclination is mainly determined by defining the maximal extent of the line at the blue side (Fig. 9), the steep drop is always at the same location for a fixed inclination and can therefore be determined almost independently of the geometry.

In summary, Fig. 9 confirms that the relativistic reflection feature is sensitive to very different parameters in the lamp post geometry. However, as soon as the primary source is not very close to the black hole or is elongated in the radial direction, the dependency on parameters such as the spin of the black hole or the incident spectrum is not large.

4.3 Implications for Measuring the Spin of a Black Hole

If we are interested in measuring the spin of the black hole by analysing the relativistic reflection, Fig. 9 and the discussion in the previous section help us to decide under which conditions we are capable in doing so. Generally, these conditions can be separated into two classes: Either the primary source is compact and very close to the black hole, or the irradiating emission comes from larger heights or an elongated structure. As has been shown in the previous section, the latter case, a large height of the primary source and an elongated structure, produces a very similar irradiation of the accretion disc and therefore we use following picture for this discussion: The base of the primary source is always located at $3 r_g$ and simply the top height of the elongated structure is allowed to change. For simplicity we call it a “compact jet” if the top height of the source is close to $3 r_g$ and “extended jet” for larger values of the top height.

As an example, Fig. 10 shows how a line profile for different spin would look like for a compact and extended jet. In the case of a compact jet the sensitivity on the spin is high (see Fig. 9d) and therefore we can clearly see the difference of a really broad line for high spin (Fig. 10a) compared to a much narrower line for low spin (Fig. 10b). If the irradiating source is elongated, the line profiles for high and low spin are very similar and especially really narrow (Fig. 10c,d). At first this underlines what was already discussed above in the context of Fig. 9: If we have an elongated source we will not be able to constrain the spin at all.

But the diagram in Fig. 10 reveals a much larger problem we have to deal with. Namely three of the four lines in the Figure will be detected as narrow lines and when fitted with any model simulating broad lines, will result in very low spin values. But this

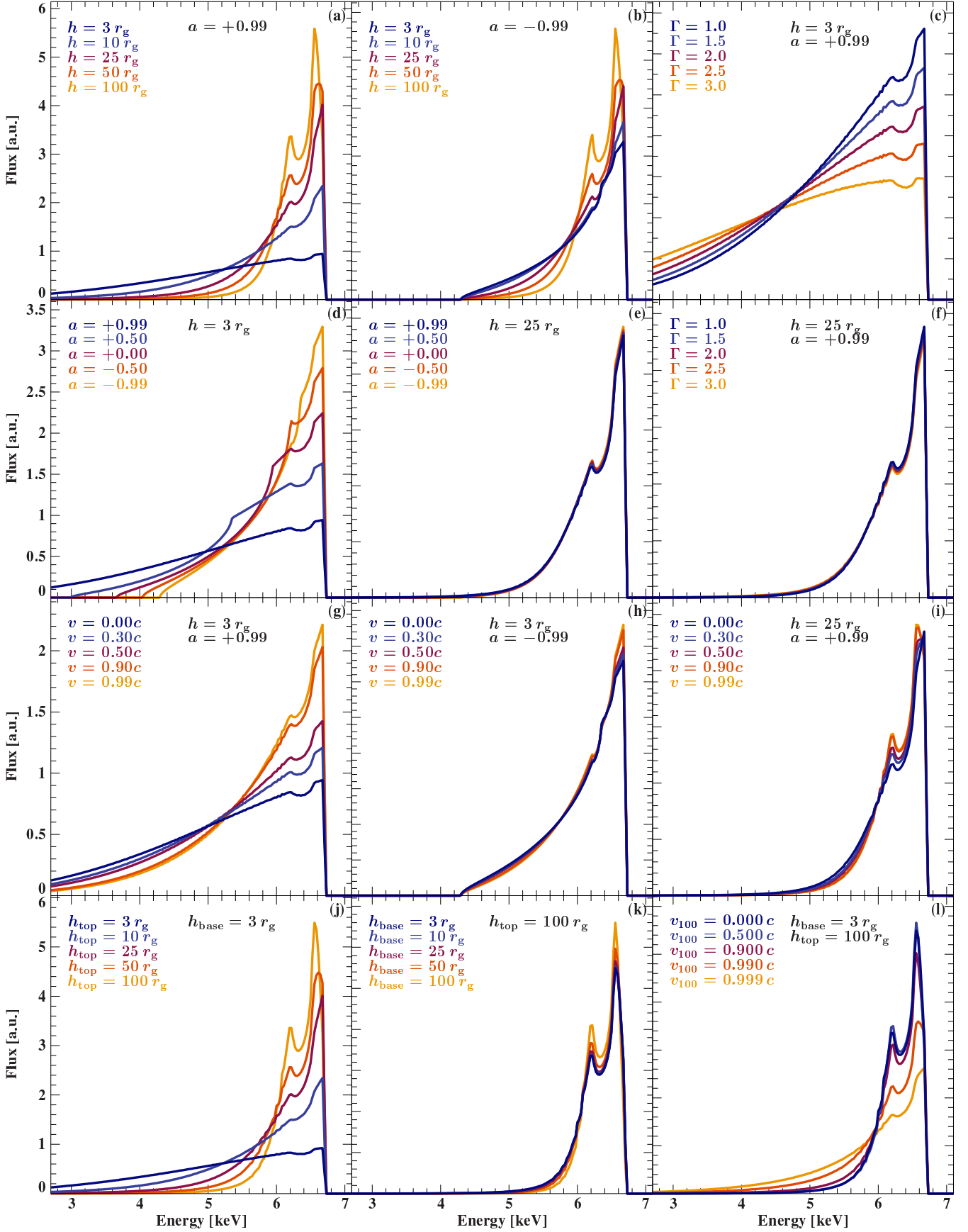


Figure 9. Line Profiles of the RELLINE_LP model for different parameters. If not stated differently we assume $a = 0.99$ and $h = 3 r_g$. We fix the inclination at $i = 30^\circ$ and the outer edge of the disc at $r_{\text{out}} = 400 r_g$. Note that we always assume that the inner edge of the accretion disc coincides with the ISCO. All profiles are normalised to have equal area, i.e., the number of photons producing each reflection feature is equal.

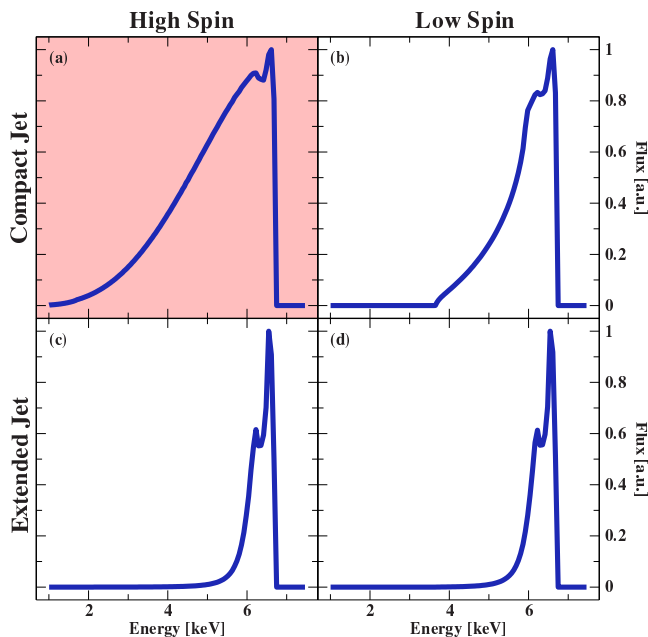


Figure 10. The line shape for combinations of high and low spin ($a = 0.99$ and $a = 0$) of the black hole and for irradiation by a compact jet (point source at $3 r_g$) and an extended jet (from $3 r_g$ to $100 r_g$). Only for high spin and a compact, low jet the line will be observed as being broad (a, red field).

would be wrong in the case of Fig. 10c, where the black hole is rapidly rotating, but only the elongated structure renders the line profile narrow. Reversing the arguments, this implies that observations of a narrow line⁶ do not allow to draw any conclusions about the spin of the black hole without knowing the exact geometry of the primary source. Hence, any black hole which was inferred to be slowly rotating due to its narrow reflection feature might very well be rapidly spinning if the accretion disc is irradiated by a source which is not compact or close to the black hole. This issue makes it even harder to produce any statistics on the spin of a sample of AGN by reflection measurements, as only from a sub-sample of the sources, the ones with relatively broad reflection features, the spin can be determined reliably.

4.4 Simulated Observations with current Instruments

We performed several simulations in order to quantify the argumentation of the previous section, on how sensitive spin measurements can be depending on the geometry of the irradiating source. For this purpose we use the EPIC-pn camera (Strüder et al. 2001) of the *XMM-Newton* satellite (Jansen et al. 2001), a widely used instrument for measuring reflection features and determining the spin of a black hole (see, e.g., Wilms et al. 2001; Fabian et al. 2009; Duro et al. 2011).

Figure 11 shows 100 ksec observations of an iron $K\alpha$ feature for different cases, namely of a compact primary source (Fig. 11a) and an elongated source (Fig. 11b), which might also be accelerating (Fig. 11c). Looking at the equivalent widths in the figure clearly reveals by what amount the line changes from a highly to a non-rotating black hole. For the canonical case (Fig. 11a), a compact primary source, there are large differences visible, already for a very

Table 1. This table summarises the result of a simulation which tries to illustrate what spin value would be obtained if we use the “standard” geometry (i.e., where the emissivity is proportional to r^{-3}) for describing data, which was originally produced by a more complex geometry of the primary source (compact, extended, or accelerated). This simulation was performed for two different values of spin ($a = 0.99$ or $a = 0.0$). All details are identical to the simulation described in Fig. 11.

	compact	extended	accelerated
$a_{\text{bh}} = 0.99$	$1.00^{+0.00}_{-0.06}$	$-1.00^{+0.27}_{-0.00}$	$0.45^{+0.24}_{-0.34}$
$a_{\text{bh}} = 0.0$	$0.2^{+0.5}_{-0.6}$	$-1.0^{+0.4}_{-0.0}$	$-0.3^{+0.4}_{-0.7}$

common exposure time of 100 ksec. On the other hand, in the case of an elongated source (Fig. 11b), there is virtually no difference at all and even future instruments will not be able to deduce the spin in this case. If the elongated source is accelerating strongly (Fig. 11c), slight differences arise (see dashed lines) but still stay deeply hidden within the uncertainties of the measurement.

As illustrated in Sect. 4.3, the inability of measuring the spin for certain geometries is not the major problem. This approach assumes that we know the geometry of the primary source, which is generally not the case. Hence, the interesting question is, what spin value would we measure when we assume the “standard” geometry ($I \propto r^{-3}$) currently used? Table 1 summarises the results of such an investigation. As expected, simulating a compact source yields nicely consistent results of spin value when fitted by the standard geometry. Contrary to that the elongated source, which produces always a very narrow line (see Fig. 11b), is always fitted by a very low spin, independent of the actual spin of the black hole. Especially for the highly rotating black hole this solution is in great contrast to the original value and even for a 100 ksec observation with current instruments far outside the confidence limits. If the elongated source now strongly accelerates while emitting photons, the obtained spin values resemble more the true ones ($a^* = 0.5^{+0.2}_{-0.3}$ for $a_{\text{bh}} = 0.99$ and $a^* = -0.3^{+0.4}_{-0.7}$ for $a_{\text{bh}} = 0.0$). However, these values still do not agree with the actual spin of the black hole. As this misleading determination of the spin does not fail due to low sensitivity but a lack of information of the primary source, future satellite missions or longer exposures will not be able to solve this issue.

5 SUMMARY AND CONCLUSIONS

In this paper we introduced a consistent approach in general relativity for modelling the irradiation of an accretion disc in the lamp post geometry. Besides a stationary point source, we also showed how a moving, a radially elongated, or an accelerating primary source can be easily modelled by slightly extending the formalism. We presented an extension to the RELLINE code (Dauser et al. 2010), which can be used for model fitting in common data analysis programs and is capable of describing all different kinds of primary sources mentioned above. We could show that only for a compact primary source the spin of the black can be measured properly. Elongated sources will always produce a narrow relativistic reflection feature. Even if we introduce a strong acceleration, the simulation showed that the spin cannot be determined for such a setup and might be confused with a compact irradiating source around a non- or negatively rotating black hole.

For the different geometries of the primary source we analysed the difference in emissivity profiles. Most importantly we

⁶ narrow, but still broad in CCD resolution

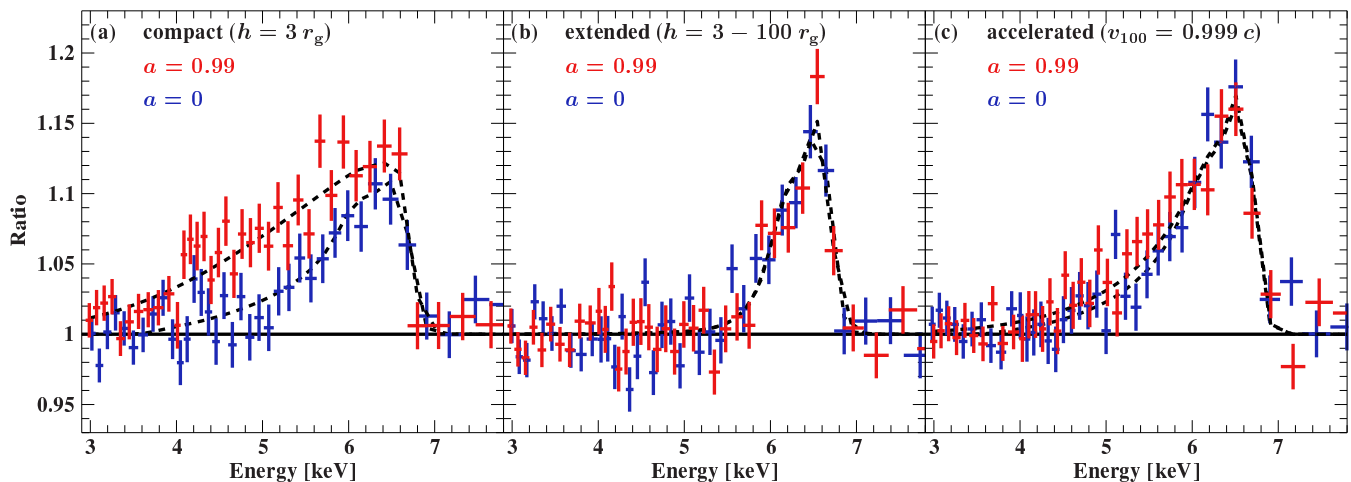


Figure 11. A simulated *XMM-Newton* observation of 100 ksec exposure of the Fe $K\alpha$ line for (a) a compact ($h = 3 r_g$), (b) an extended jet ($h = 3-100 r_g$), and (c) again an extended jet ($h = 3-100 r_g$), but now extremely accelerated such that $v_{100} = 0.999 c$. Parameters were taken to be similar to MCG-6-30-15 in the *XMM-Newton* observation with Obs. ID 0111570101 (Wilms et al. 2001). In order to obtain consistent line strengths we fixed the desired geometry and spin value and fitted the normalisation of the line to the data. The equivalent widths for high spin compared to non-spinning were determined to 432 eV vs. 175 eV (a), 110 eV vs. 109 eV (b), and 255 eV vs. 208 eV (c).

could see that the innermost part (i.e., $r < 2 r_g$), which is also the steepest part of the profile, is completely independent of the geometry and movement of the source (see also Fukumura & Kazanas 2007). Hence, by changing the geometry in our lamp post setup, we will not be able to create steeper profiles than the standard one for a point source in order to explain the really steep emissivity indices measured in many observations (see, e.g., Wilms et al. 2001; Fabian et al. 2002; Fabian et al. 2004; Brenneman & Reynolds 2006; Dauser et al. 2012). Even sources extended perpendicular to the rotational axis seem not to be able to alter the steepness of the emissivity (Wilkins & Fabian 2012). However, these steep emissivities might be artificially created due to an over-simplified modelling. Recent analyses show that describing the reflection at the accretion disc with a constant ionisation, instead of an ionisation gradient that would be expected from basic arguments (using, e.g., Shakura & Sunyaev 1973), would result in over-estimating the emissivity index (Svoboda et al. 2012). Moreover we showed that this independence of the inner part of the emissivity profile from the geometry comes from the fact that this steepening is mainly due to the strong blue-shift the photons exhibit during their flight towards the accretion disc. Although the strong focusing of the photons close to the black hole is commonly used to explain this steepening, Fig. 2 evidently showed that this effect actually only contributes a minor part to the steepness compared to the energy shift.

Using the new `RELLINE_LP` model, a variety of broad emission lines for very different parameter combinations was produced. This showed that only compact sources close to the black hole produce the really broad lines and are sensitive to parameters like the black hole spin or the steepness of the incident spectrum. Hence, observing a broad line already strongly constrains the primary emission region to be very close to the black hole, which also implies that most of the reflected radiation originates from the innermost parts of the accretion disc. Moreover the narrow line profile created by an elongated primary source is very similar and therefore hardly distinguishable from a point source at an effective, intermediate height. Introducing an acceleration of the elongated source effectively reduces the flux from the upper part of the source. This ac-

celeration leads to a broadening of the line shape, which means that it becomes again more sensitive to the black hole parameters and more similar to the line profile created by a compact source at low height.

Simulating observations with current instruments, the line profile of elongated sources will always be rather narrow. Applying standard line models to X-ray spectra of such systems will always result in a low spin of the black hole, although this might not be the actual case. We therefore conclude that only a really broad emission line will give us the possibility to determine the spin of the black hole unambiguously. Such systems must have a compact emitting source and a rapidly rotating black hole. In all other cases with narrower lines, the emission line can either be described by a slower rotating black hole or an elongated source, which could have any arbitrary spin. The thought that a narrow, relativistic reflection feature might be produced by a primary source at a larger distance to the black hole not new. Vaughan et al. (2004) suggested such a geometry to explain the reflection feature in Akn 120, as it required a flat emissivity at the inner edge of the accretion disc. In agreement with our results presented above, Chiang et al. (2012) conclude for XTE J1652-453 that the spin cannot be constrained if the primary source is assumed to be located at a large height above the black hole.

We note that in some sources broad lines have been seen to be changing. Galactic black holes are seen in two states, a hard state where the X-ray continuum is a power-law with an exponential cutoff that can be well described by a Comptonization continuum with (relativistically smeared) reflection, and a soft state in which thermal emission from an accretion disc dominates the emission (see, e.g., McClintock & Remillard 2006, for a review). During the hard state a relativistic outflow is present in these systems, which is quenched during the soft state (Fender, Belloni & Gallo 2004). This jet can carry a significant percentage of the total accreted power (Russell et al. 2007, and references therein). In many models for the black hole states it is assumed that the inner disc radius is receded outwards during the hard state and that it only extends to the ISCO during the soft state (e.g., Esin, McClintock & Narayan 1997; Dove et al. 1997; Fender, Belloni & Gallo 2004). If the

Comptonizing plasma is located close to the accretion disc, an inward moving accretion disc would increase Compton cooling of the Comptonizing plasma, which then collapses, resulting in the soft state. One of the major observational arguments for this picture are changes of the relativistically broadened line in these systems. For example, the Galactic black hole GX 339–4 is seen to have a broad line during the more luminous phases of the hard state, i.e., closer to the soft state (Miller et al. 2008, but note Kolehmainen, Done & Díaz Trigo 2011) and a narrow line during the fainter phases of the hard state (Tomsick et al. 2008, see also Nowak, Wilms & Dove 2002). While this behaviour is consistent with a receding disc, given the discussion above this picture is not as clear cut anymore. The broad band radio to X-ray spectra of galactic black holes are also consistent with emission from the base of a jet, with a statistical quality comparable to that to the disc-corona models (Markoff, Nowak & Wilms 2005; Nowak et al. 2011). Furthermore, the jet properties in these sources are observed to change as the state approaches the intermediate and soft states, including the observation of single radio blob ejections close to transitional states (Fender et al. 2006; Wilms et al. 2007; Zdziarski et al. 2011). If this is the case, then according to the discussion above the change in line width from narrow to broad seen in GX 339–4 could also be due to a change in the jet properties from an extended jet far away from the soft state towards a compact jet, rather than a change in the inner disc radius (see also Miller et al. 2012).

For observations of sources where the line shape changes only on very long timescales, such as AGN, our results imply that solely relying on spectral fits we are only able to measure the spin of rapidly rotating black holes. All other spin values obtained from fitting reflection spectra are not trustworthy, as they can be easily explained by changing the geometry of the irradiating source. This result is of significant importance, e.g., when trying to use spectral black hole spin measurements to infer the spin distribution of the local black hole population, as it introduces a significant bias towards sources with small jet sizes.

Fortunately, however, reflection measurements are only one of several independent ways to determine black hole spins. For Galactic sources, where the accretion disc temperature is large enough that it can be observed in the soft X-rays, it is possible to use the shape of the accretion disc spectrum to infer the black hole spin (McClintock et al. 2010, but see Dexter & Quataert 2012). Where a broad line is observed, the results of continuum spin measurements and line measurements agree (e.g., Gou et al. 2011 and Duro et al. 2011 for the case of Cyg X-1). Furthermore, X-ray reverberation techniques allow constraining the location of the primary source through measuring X-ray time lags between the continuum and the reflection spectrum (Stella 1990; Matt & Perola 1992; Reynolds et al. 1999; Poutanen 2002; Uttley et al. 2011, and references therein). Despite the comparatively small effective area of current X-ray instruments, such measurements have already been successfully applied to a small subset of extragalactic black holes, e.g., 1H0707–495 (Fabian et al. 2009; Zoghbi et al. 2010; Zoghbi & Fabian 2011; Kara et al. 2013), NGC 4151 (Zoghbi et al. 2012), MCG–6-30-15 and Mrk 766 (Emmanoulopoulos, McHardy & Papadakis 2011), and PG 1211+143 (de Marco et al. 2011)). In all these measurements the primary source was constrained to be compact, extending only in the order of a few-to-tens r_g away from the black hole. Current concepts for the next generation of X-ray satellites such as the Large area Observatory for X-ray Timing (*LOFT*; Feroci et al. 2012) or the Advanced Telescope for High Energy Astrophysics

(*ATHENA*; Barcons et al. 2012) all have effective areas of several square meters. With such large collecting areas it will be possible to measure high-quality X-ray spectra for a larger sample of AGN within the typical light travel timescales for the lamp post model. These facilities will allow X-ray reverberation measurements to constrain the emission geometry of the observed sources such that the degeneracies pointed out above can be broken for a larger sample of sources.

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