Soap Films as 1D waveguides

Article *in* Optofluidics Microfluidics and Nanofluidics · May 2014

DOI: 10.2478/optof-2014-0002

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DE GRUYTER OPEN Optofluid. Microfluid. Nanofluid. 2014; 1:27–33

Research Article **Open Access**

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Soap films as 1D waveguides

Abstract: Laser light is injected in a free standing horizontal draining soap film through the glass frame sustaining the film. Two propagation regimes are clearly identified depending on the film thickness. At the beginning of the drainage, the soap film behaves as a multimode-one dimensional optofluidic waveguide. In particular, we observe that the injected light creates a bottleneck in the film and part of the injected light is refracted leading to whiskers. At the end of the drainage where the film thickness is below 1μm, there is a strong selection among the various possible optical modes in the film, and part of the light is deflected. This leads to a self selection of the mode propagation inside the film.

Keywords: optofluidic waveguide, laser injection, soap film thinning, thin film deformation

DOI 10.2478/optof-2014-0002
Received May 6, 2014; accepted August 18, 2014

1 Introduction

Optofluidics is a research domain that takes advantages of microfluidics and optics to synthesize novel functionalities for applications that include biophotonic systems, lab-on-chip devices, biosensors and molecular imaging. This term has only very recently been used for the first time [1–3]. However, one of the first optofluidic waveguide with fluid core/fluid cladding was reported in the nineteenth century in which Jean Daniel Coladon injected light in a water jet flowing from a hole in a water tank [4, 5]. The light was then guided along the water jet. Curiously, most of the recent experiments using optofluidic waveguides used the same experimental characteristics, i.e. flowing fluids with different indexes (i.e. different fluids, or the same fluid with different temperatures) and rather larger waveguides with multimode propagation [6–8]. Monomode propagation has been barely reported [9] and deduced from index difference argument [10]. However, monomode propagation could strongly enhanced the biodetection and chemical efficiency capacities of liquid waveguides [11, 12]. Moreover, since most of the optofluidic waveguides take advantages of the possibility of the waveguide to be reconfigured or adapted continuously in ways that are not possible with usual optical waveguides [9, 13], the investigation of the inner mechanisms of the light-liquid interaction is necessary. One can then wonder how the light would be guided when the waveguide becomes thinner and thinner along propagation and whether the presence of light could modify the waveguide properties. The aim of our work is to consider a draining soap film as a one dimensional model waveguide [14], with strong optical confinement [15] to experimentally investigate the propagation inside the film as its gets thinner during the drainage.

The manuscript organization is as follow. In the next section we present our experimental set up, describing the lasers we used and the soap solution. We then detail our experimental observation together with the results, highlighting the different propagation regime we see at the beginning and at the end of the drainage. In the following section we discuss about what we call "whiskers" and about deflection. In the last section we question ourselves about monomode propagation, before reaching the conclusion.

2 Experimental set up

The experimental set up that is depicted in figure 1. The laser light from a solid state laser (Crystal Laser, λ = 532 nm, w = 360 μm, output power attenuated to P = 2 mW in order to prevent heating) is injected in an horizontal free standing draining film. The diameter of the toroidal glass frame is 4 cm and the diameter of the glass rod is 4 mm. The soap solution consists of 5.4 % sodium lauryldioxyethylene sulphate (SLES, 55.6 g/L, Cognis) in pure water. The refractive index n has the same value for the film and for the bulk solution. Actually, the variation of the refractive index n during the film drainage is negligible. A difference exists between the interface saturated by surfactants and inside the film [16] but the interfacial

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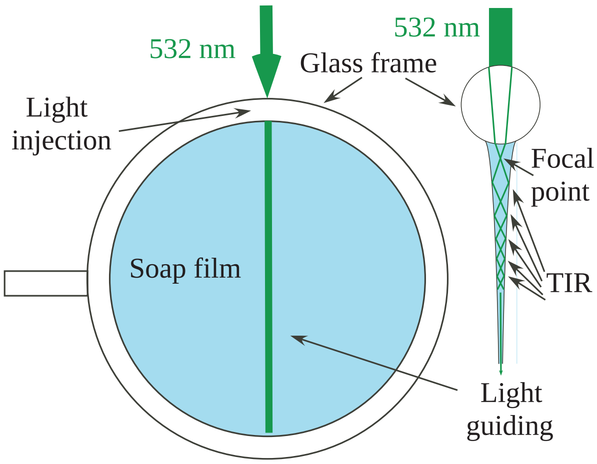

Figure 1: Left side: injection of the laser beam from the side of the frame. Right side: zoom of the injection showing the focalization and the guiding of the light. TIR: Total Internal Reflection.

thickness which is of about 2 nm is too small to have any influence here. The soap film, first formed into a metal ring, is placed gently into the horizontal fixed toroidal glass frame. All experiments are performed at a controlled temperature $T = 20.0 \pm 0.5°C$. Besides, the glass frame is cleaned with Hellmanex (Sigma-Aldrich, diluted 10 times in pure water), acetone (99.5 %), ethanol (99%), and rinsed with pure water after each experiment in order to remove dust particles and soap film pollution.

We inject the laser from the side through the toroidal glass frame. Actually it acts as a cylindrical lens and the laser beam is thus focused in the soap film at a 1.52 mm distance from the frame. The distance has been calculated using the propagation of gaussian beams [17]. To calculate the beam waist inside the film, we start from the size of the waist of the laser. Then, taking into account the free space propagation, the curvature of the glass rod and the free space propagation in water, we use the so called ABCD matrices to evaluate the size of the focalized beam. We estimate the spot size in the vertical direction to be 2 μm. The beam then undergoes several total internal reflections and is then guided into the soap film like in a funnel. We estimate that the injected laser power into the film is between 50 and 80% of the initial power, depending on the film thickness and on the various effects that will be discussed in the next section. The polarization of the injected beam is imposed by a half wave plate (choosing either a TM or a TE polarization). We are also able to probe the thickness of the film using a home built optical interferometer [18]. We use the interferences from two low power He-Ne lasers ($\lambda = 543.5$ nm and 632.8 nm, CVI Melles Griot) to measure, via the extrema of the transmissions, the film thickness. The absolute precision on the thickness measurement is of the order of 10 nm.

3 Observations

We record by a webcam on a computer the film behavior from its formation, until it breaks. We can then easily follow the guiding of light inside the soap film versus time. One can clearly identify two regimes. The first one, at the beginning of the drainage shows a straight propagation of light and the apparitions of "whiskers" (see figure 2). The second regime at the end of the drainage corresponds to part of the light traveling straight and part of the light that is deflected (see figure 3).

3.1 Beginning of the drainage

Let us start with the beginning of the drainage and the appearance of the so called "whiskers" (see figure 2). It seems that these whiskers are clearly emerging closely from the point where the light is injected in the film. They resemble a sweeper or whiskers that appear in or on the soap film. These whiskers have already been observed [19, 20]. They are rather unstable and jump from one side of the direction of the injection of the laser to the other in a random way. However, the light from theses whiskers stops somewhere in the middle of the film and is not emerging from the frame. It seems that this light is not guided inside the film. The whiskers are more visible with a TE polarization when looking to the film from the top than for a TM polarization. The effects of the two polarizations are reversed when looking at a grazing incidence.

Curiously, the laser light coming out of the frame that corresponds to a straight propagation, cannot be seen on the soap film in general (see figure 2). This means that this light is very little diffracted or diffused inside the soap film. This is an evidence that the air/liquid interfaces of the soap film are very smooth and that the roughness of the limiting surface of the waveguide is very low.

3.2 End of the drainage

The situation is very different at the end of the drainage, as can be seen in figure 3. First of all, there are two or more output beams now. There is a kind of filamentation of the light beam. Besides one can now clearly follow the light trajectories inside the film. The light propagation must have been perturbed, whereas the interface rough-

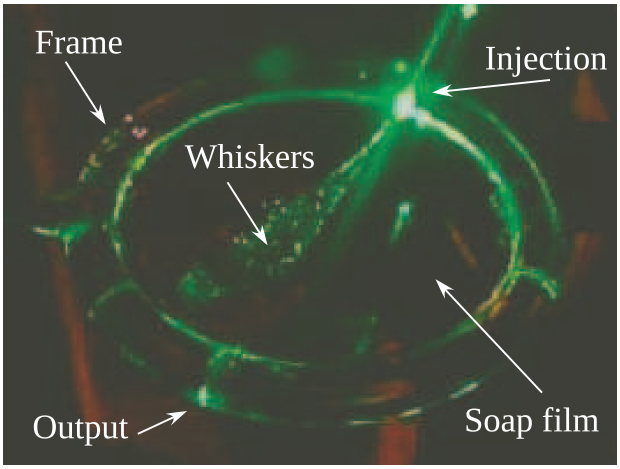

Figure 2: Guiding of the light inside the soap film at the beginning of the drainage. One can see "whiskers" appearing, originating from the injection point.

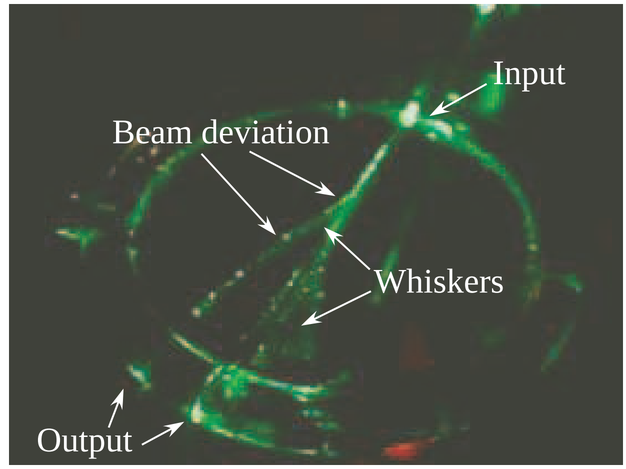

Figure 3: Guiding of the light at the end of the drainage. The laser beam is split in several parts and deflected.

ness should still be very low. One can see in figure 3 that part of the beam has beam deflected, somewhere in the middle of the soap film.

The beam is here split in two parts. One part still travels along a straight line in the direction of the injected light, whereas the deflected beam may also encounter one or more deviations during propagation. Actually these deflections are random and can occur on both side from the direction of the laser injection. There are also very unstable, with a time constant of the order of 0.1 s. They become even more unstable as the film drains. We have observed up to 3 output beams. At the end of the drainage, the beam hardly propagates inside the film. It seems to be subjected to high losses, since the beam propagation becomes more and more visible.

One can also note that the whiskers do not originate from the input of the laser beam any more. They are even sometimes not present. When they are present, the occurrence is at a point where the beam is split or deflected. There is no light emerging from the frame that corresponds to these whiskers. As for the beginning of the drainage, the observation of the effect is more pronounced for a TE polarization than for a TM polarization.

4 Discussion

4.1 Origin of the whiskers

As already mentioned, whiskers in the propagation of light in soap films have already been reported [19, 20]. In those articles, their origin was explained as being due to nonlinear effects appearing inside the soap film in the presence of light. Whereas nonlinear effects and self focalization may occur in soap films [21], we think that the origin is not from non-linearities. Indeed we use a low power laser, and although the light is focalized, the frame acts as a cylindrical lens and the light is focalized in one dimension only. For lower laser power, the time of occurrence of the whiskers diminishes, for higher optical powers, the nonlinear effects could not be excluded. Our working conditions result in a kind of compromise between these two effects.

We rather think that the origin comes from a recently reported effect, due to total internal reflection and spatial shift at the interface, also called Goos-Hänchen shift [22]. Actually, this spatial shift, combined with the gaussian structure of the beam leads to a force perpendicular to the interface that creates a dip in the interface. This effect can also be seen as being due to the evanescent wave whose strong light intensity gradient attracts the air molecules around it and traps them close to the interface. This creates an over pressure in the region where the optical beam is totally reflected, leading to this dip. Of course, as the angle of incidence gets closer to the critical angle $i_c = 48.75°$, the evanescent field is stronger and its penetration depth in air is higher, leading to a stronger effect.

In our case, close to the focal point (about 1.5 mm from the frame), the film thickness is not uniform and the film gets thicker as we approach the peripheral meniscus. Then part of the light impinges on the air/liquid interface close to the critical angle. This leads to a force that induces a dip at the interface. This dip is not symmetrical since the curvature of the interface is not uniform, and since the light close to the critical angle is also close to the frame. Then the light impinging on the interface at the dip location is

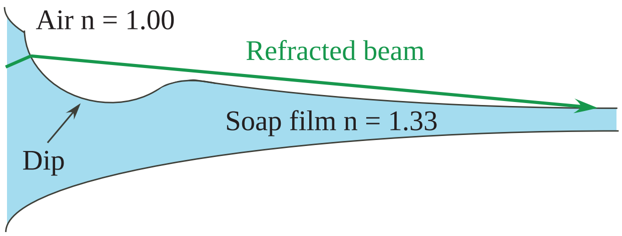

Figure 4: Light is refracted at the dip position and hits the film further where it is diffused. The interface deformation and the angle of incidence of the beam are exaggerated on the figure.

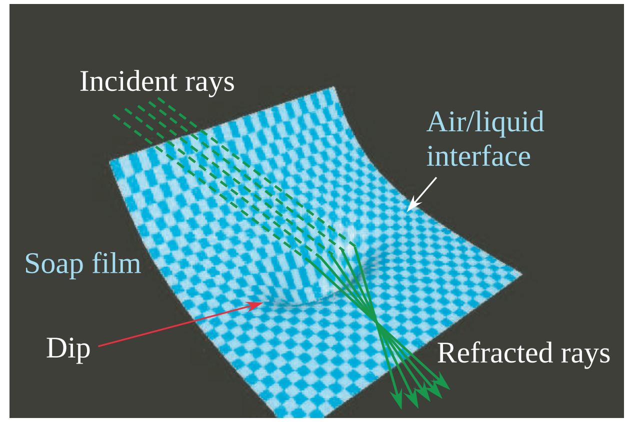

Figure 5: Air/liquid interface in 2 dimensions showing the dip in the film. Some incident rays are refracted.

not in total reflection any more and gets refracted according to the well-known Snell's laws of refraction (see figure 4). It then hits the film further, far from the meniscus. This is the reason why, at the beginning of the drainage, the whiskers seem to originate from the focal point.

This dip should have an ovoid shape (see figure 5). Indeed, the frame acts as a cylindrical lens. Then close to the laser focus, the laser should have a cigar like shape. Besides, since the depth of the deep depends on the intensity and since the laser is gaussian, the depth of the deep should be more pronounced in the center of the beam than on the edges. Then considering several incident rays, the corresponding refracted beam hits the air/liquid interface further and gets diffused, thus creating an illuminated surface that resembles a sweeper or whiskers. Actually these whiskers originate from rays that have been propagating in air and that get diffused when they hit the air/liquid interface. They thus cannot be out coupled from the frame. When the light is TM polarized, since the polarization is conserved during diffusion processes, the whiskers are hardly noticed when looking at the film from above. Conversely, when the laser is TE polarized, there is more diffusion. The conclusions are reversed when looking at grazing incidence. It is thus difficult to estimate the ratio of the visibility of the whiskers for the two polarizations in this way. However, the time of occurrence of the whiskers is less with a TE polarization since the penetration depth of the evanescent wave is smaller for a TE polarization then for a TM polarization.

This dip is unstable since the light responsible for the deformation is in total internal reflection close to the critical angle. As the interface deforms, those rays are not in total internal reflection any more. They are diffracted. Then the force on the interface is partly released. The dip then slightly moves. The refracted light as well as the corresponding light that has been partially reflected is no more guided and escape from the film. The whiskers have no direct link with the guiding of light.

4.2 Origin of the deflection

Let us now move to the end of the drainage, when the film gets thinner. In this particular configuration, the light beam could be split and deflected in the soap film during propagation. Actually this corresponds to guided light since there is some light out coupled from the frame at the end of the propagation (see figure 3). There is also light that still propagates along a straight line corresponding to the direction of the injection. For the same reason as the one previously mentioned we do not think that this is due to purely non-linear effects. Our belief is that the deflection is due to a selection between the different modes that propagate inside the film due to the film thinning.

We have thus investigated the drainage (see figure 6) of the soap film using the technique described in the first paragraph using interferences in transmission [18]. We have measured the film thickness in 3 different points. The first one referred as to point 1 is in the middle of the frame, the second one referred as to point 2 is 3 mm away from the frame, in the direction of the injection, close to the focal point of the injected laser. Point 3 is outside the direction of injection in between points 1 and 2. The arrows on the figure indicate when the pictures of figures 2 and 3 have been taken.

First, whereas the thickness of the film at point 2 is of the order of several micrometers for figure 2 (corresponding to 35 s after the beginning of the drainage), the thickness decreases to 750 nm for figure 3 (corresponding to a time of 98 s). At the beginning of the drainage, the thickness is higher than the size of the focal point. Using a ray-picture of the propagation of light, there is thus light impinging on the interface under total internal conditions, close to the critical angle, leading to the dip in the film and to the whiskers. However, at the end of the drainage,

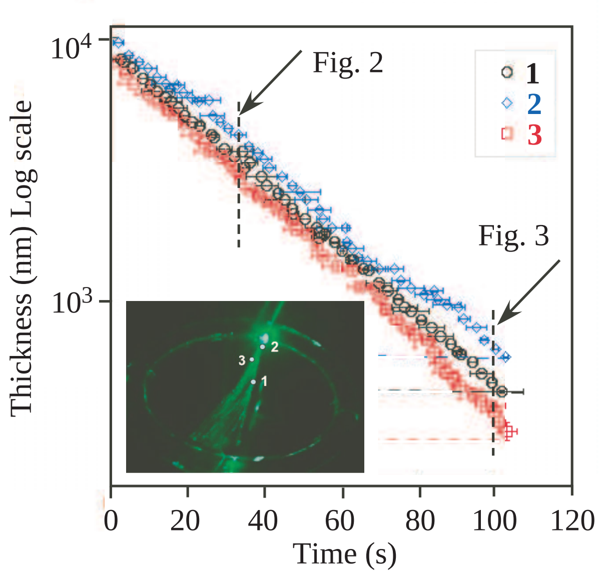

Figure 6: Thickness of the soap film measured at three different points. These points are shown on the picture of the film, in the inset of the figure. The arrows indicate the time when the pictures of figures 2 and 3 have been taken.

the thickness of the film is smaller than the size of the focal point. The light is already guided in the film. The ray picture cannot be used any more. There is no dip and less loses at this point. This explains why there is no more whiskers at the end of the drainage close to the focal point.

Second, whiskers sometimes appear during propagation at a point where part of the light is reflected. These whiskers should have the same origin as the ones at the beginning of the drainage. The underlying mechanism should be that, as the film is getting thinner, more an more optical power must be present in the evanescence of the guided mode. There must be a dip appearing due to the light in the evanescent wave. This dip thus refracts part of the light and may also deflect other parts. Actually, one may wonder whether the soap film behaves as a multimode or as a monomode one dimensional guide as the film gets thinner.

5 Monomode or multimode guide?

Considering a given waveguide, one can define a cutoff wavelength λ_c. It is the maximum wavelength that propagates in a waveguide [10]. In a rectangular metallic wave guide with typical length a and b, with an inner index of refraction $n = 1.33$ the boundaries conditions imply [23] that the cutoff wavelength of a mode with transverse orders m and p writes

$$\frac{1}{\lambda_{c(m,p)}} = \frac{1}{n}\sqrt{\left\{\frac{m}{2a}\right\}^2 + \left\{\frac{p}{2b}\right\}^2}, \tag{1}$$

then, for a one dimensional waveguide or slab waveguide, since b tends toward infinity, the previous expression is modified into

$$\frac{1}{\lambda_{c(m)}} = \frac{1}{n}\frac{m}{2a}, \tag{2}$$

In a dielectric slab waveguide like the soap film we consider here, due to the existence of an evanescent wave, the boundary conditions are slightly modified. Then equation 2 is modified into [10, 24, 25]

$$\frac{1}{\lambda_{c(m)}} \approx \frac{m}{2a\sqrt{n^2-1}}, \tag{3}$$

assuming a guiding between a medium with index n and air.

The problem we address here is slightly different. We are not looking at the maximum wavelength that can propagate. However, for a given wavelength equal here to λ = 532 nm, one can define a minimum thickness, or cutoff thickness a_c for the mode $m > 1$ to propagates

$$a_{c(m)} = \frac{m\lambda}{2\sqrt{n^2-1}}, \tag{4}$$

Then, the soap film would be monomode for a thickness $a < a_{c(1)}$ with $a_{c(1)} = 303$ nm, for 303 nm $< a <$ 606 nm, there would be 2 modes inside the soap film that propagate, for 606 nm $< a <$ 909 nm, there would be 3 mode, and so on. Clearly, in figure 3, which corresponds to a thickness indicated by the second arrow of figure 6, the film thickness is below 1 μm. There is between 2 and 3 modes propagating inside the film at point 2. There is between 1 and 2 modes at point 1 (see figure 6). Then the first point of deviation of figure 3, is a point where the thinning of the film leads to a thickness nearly above 300 nm. There is more and more light in the evanescent wave.

Thus a dip is starting in this region, and the second order mode propagating inside the film, which corresponds to a mode with no light power in the center, gets deflected by the dip. There is also some light refracted leading to the whiskers. As the deflected beam travels further, the film may get thinner, leading to a dip where the beam is again deflected. The second order mode makes a kind of random walk, whereas the fundamental mode travels straight. Note, that in the case of a one dimensional slab waveguide, there is always at least one propagating mode whatever the thickness [25]. However, this mode may encounter quite high losses. It may also induce a rupture in the soap film in this region.

According to the previous paragraph, when the thickness is of the order of a fraction of micrometer, the soap film is able to self-select the fundamental propagating mode and to deflect the others. This phenomenon resembles the self selection or self adaptation occurring in mode propagation in radiation pressure induced waveguide [26, 27], although in their case, the laser propagates in the lower index medium.

Our optofluidic waveguide is quite a high contrast waveguide since the difference in optical index $n_{water} - n_{air}$ equals 0.33. Besides, the photoelastic coefficients of the air/liquid interface are rather low compared with the photoelastic coefficient of solid core waveguides. One may then wonder whether the electrostrictive force could be responsible for such an observation [28]. However, in our case, the appearance of the deflection phenomenon is polarization independent, only the magnitude of the effect and the contrast of the observation depends on the polarization. Thus an electrostritive origin of the effect can be eliminated.

The mechanism responsible for the dip is a kind of non-linear effect. Indeed, when the film gets thinner, close to a cutoff thickness for a given mode, the intensity in the evanescent wave gets higher. This produces a force on the film in this region that induces a dip. This reduces the film thickness, allowing even more light intensity in the evanescent wave. This non-linearity is however different from purely non-linear effects that may arise in total reflection conditions which are due to a change of the optical index [29]. Besides, since the time constant of the film response is of the order of several ms, this non-linear effect is quite slow, compared with pure non-linear effects. Besides, this appearance of the dip inside the film could be a way to change or to regulate the rate flow within waveguides using flowing liquids [13] or in optofluidic micro pumps [30].

6 Conclusion

In this article we have discussed the guiding of laser light in an horizontal free standing draining soap film. We clearly identified two regimes. The first one corresponds to the beginning of the drainage where the film thickness is over 1μm. This regime is characterized by the occurrence of whiskers that originate near the injection of the light in the film and which is due to the force exerted by the evanescent light on the film. The second regime appears at the end of the drainage where the film thickness is below 1μm. There is a self selection of the mode propagation inside the film, the higher modes being deflected in a random way, leading to monomode propagation. This fundamental mode always travels according to a straight line.

This self mode selection relies on the force exerted by the evanescent wave on the film that leads to its thinning. One may then wonder whether such self mode selection could also appear in optofluidic waveguides with gain inside [31–33] or in whispering gallery mode lasers in micro droplets [34–36]. It would thus be a way to self tune or to control optofluidic lasers to make same single mode. It could also help to enhance the fluorescence [37] or the sensing power of the evanescent light [38, 39] in sensors using waveguide detection. It may also improve the efficiency of biological analysis [40, 41].

Acknowledgement: This work has been performed within the European COST action MP 1205 Advances in optofluidics: integration of optical control and photonics with microfluidics. We would like to thank J. P. Delville (LOMA, Bordeaux) for fruitful discussions.

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