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A lot of good physics in the Cartesian diver

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Abstract

We present a simple mathematical analysis for deriving the pressure and temperature variations for complete sinking in the Cartesian diver experiment. Some additional remarks on the system, with respect to the existing literature, are also made.

Introduction

The Cartesian diver experiment certainly occupies a place of honour in old physics textbooks [1, 2] as a vivid demonstration of Archimedes' buoyancy. The experimental apparatus is nowadays presented in physics apparatus catalogues as a significant classroom experiment [3]. The diver is a little glass imp partially filled with water immersed in the same liquid contained in a glass cylinder. A hole in the diver allows exchange of water from the container to the diver, or vice versa. The top of the container is covered with a rubber diaphragm. Pressing down on the diaphragm, the diver sinks because water enters in. The original experiment, as described in old textbooks, shows Archimedes buoyancy qualitatively: when the increased weight of the diver is not counterbalanced by Archimedes' buoyancy, the diver sinks. When the pressure on the diaphragm is relieved, compressed air in the diver expels water from the diver, so that the latter may rise up again. For pressure changes, Δp , above a certain critical value, Δp_c , irreversible sinking is obtained.

This device has received renewed interest in the pedagogical physics literature [4–10]. Nevertheless, no substantial novelty has been added, except for an in-depth mathematical analysis given by Güémez *et al* [11]. We note a beautiful booklet by Siddons [12] outlining all the physical features involved in the Cartesian diver.

Here we propose a rather simple mathematical analysis of the Cartesian diver, considering it sinking just below the water surface. We briefly comment on some simple classroom experiments that can be performed to qualitatively understand the phenomenon and give an account of the equivalence of the effect of two thermodynamic variables on the system's properties; namely, the pressure p and absolute temperature T . By this analysis a simple expression for the temperature and pressure variations for complete sinking of the diver are obtained. Some remarks on irreversible sinking of divers in small containers are made.

Observed phenomenology

For simplicity, we take our Cartesian diver system to consist of a glass test tube filled with water (as the container) and a diver constructed from a thin glass eyedropper. The eyedropper is cut to a length of about 2 cm and closed, by means of a gas flame, at the cut end. By introducing a small amount of water into the diver with a syringe, the diver floats with a negligible volume above the water surface, as shown in figure 1. In this situation the small pressure exerted by a finger closing the top of the test tube causes sinking of the diver, which is easily visible in a school classroom (figure 2). Releasing the finger from the top of the test tube, the diver rises up again. Here, we are still not



Figure 1. The diver ‘just floats’ on the water surface. The pressure is equal to the atmospheric pressure (the finger is not closing the test tube).



Figure 2. The diver sinks below the water surface when the pressure is increased by simply closing the test tube with a finger.

concerned with irreversible sinking and, for the moment make no mention of this phenomenon. Pedagogically significant observations at this stage often come from attentive students. Students may ask ‘*What would happen to the diver if we assumed no adhesion between the water and the inner walls of the eyedropper?*’ At this point we note that no paper referenced outlines the role of adhesion of water to the walls of the diver. In our opinion, the diver acquires greater weight because the water inside *adheres* to its inner walls. In this way the weight of the diver is given by the weights of the water inside and the weight of its glass portion, the weight of the air trapped inside being negligible.

As for irreversible sinking, a longer column of water is needed and the same experiment performed with the diver. We use, for this purpose, a glass tube of about 1.2 m in length. The tube, of diameter 1.5–2 cm, is closed at the bottom with a rubber cork. The pressure of a finger on the top of the tube is sufficient, even in this case, to cause an increase in pressure for sinking. When the diver starts sinking in the column, it may reach higher values of h below the water surface and may thus go towards irreversible sinking when the critical value $h = h_c$ is reached. The approximate value

of h_c can be seen to depend on the ratio between the diver’s volume V_{ext} emerging out of the water surface before exerting additional pressure on the top and the value of the initial bubble of air in the diver V_0 . We note that, for $\frac{V_{\text{ext}}}{V_0} \approx 0.1$, the value of h_c is about 1 m, so that irreversible sinking is observable in a glass tube of about 1.2 m. On the other hand, if the diver *just sinks* (i.e. its upper part is initially just at the water surface, so that $\frac{V_{\text{ext}}}{V_0} \approx 0.01$) the level beneath the free surface for an irreversible sinking is of the order of a few centimetres. An instructor, with a little practice, can thus realize a ‘sensitive’ diver, irreversibly sinking in an ordinary test tube, as shown in figures 3 and 4.

Other interesting observations can be proposed to students. For example, we could realize a *just sinking* diver (as before) on a low atmospheric pressure day. We could therefore find, days after, that, because of an atmospheric pressure increase, the diver irreversibly sinks. The Cartesian diver can thus work as a sensitive baroscope. Explanation of this phenomenon can be obvious, after the students have qualitatively understood the mechanical response of the diver to a pressure change.



Figure 3. The diver ‘just sinks’ (i.e. it is floating just at the water surface in the test tube).

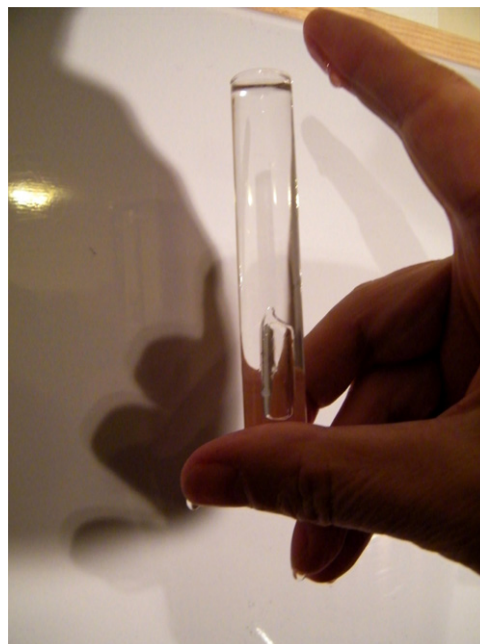


Figure 4. Starting from a just sinking condition, the diver sinks irreversibly in a test tube when the pressure is increased by closing the top of the test tube with a finger.

Another important point could be the mechanical response of the diver to a temperature change. Consider again a test tube to be the container in our Cartesian diver system. Consider a diver floating with non-negligible volume V_{ext} above the free water surface. This type of diver does not sink completely simply by closing up the test tube with a finger. When placed in the ice box of a refrigerator, after about half an hour we find that the water inside the test tube has turned into an ice column and that the diver has sunk. When the ice melts and the water temperature rises to room temperature, the sinking of the diver is found to be reversible. An attentive student may observe, in this last experiment, that the ice column is some millimetres above the rim of the test tube, provided the water level in the test tube was *just at the rim* at room temperature.

Simple analytic description of the observed phenomena: pressure change

After having made the simple experiments and stimulated observations by the class, a problem-solving session involving elementary physics concepts could follow. We refer to figure 5, which shows the Cartesian diver schematic. Let V_T be the

total volume of the diver, V_{ext} the volume emerging from the water surface at atmospheric pressure p_a , m_i the water, of density ρ , inside the diver, m_g the mass of the diver’s glass and g the acceleration due to gravity. Neglecting the mass of air trapped in the diver and the buoyancy on the glass volume, the static equilibrium equation is written as

$$(m_g + m_i)g = \rho(V_T - V_{\text{ext}})g. \quad (1)$$

Because $m_i = \rho(V_T - V)$, where V is the volume of air inside the diver at pressure p , which takes account also of hydrostatic pressure at the bottom of the diver, we can write equation (1), for any pressure p at the top of the container as

$$\rho(V - V'_{\text{ext}}) = m_g, \quad (2)$$

where V'_{ext} is the volume of the diver above the water surface. Let us now assume that a pressure variation dp is applied to the system isothermally. Treating the air as a perfect gas, we may write $pV = \text{constant}$, so that $p dV + V dp = 0$ and

$$dV = -\frac{dp}{p} V. \quad (3)$$

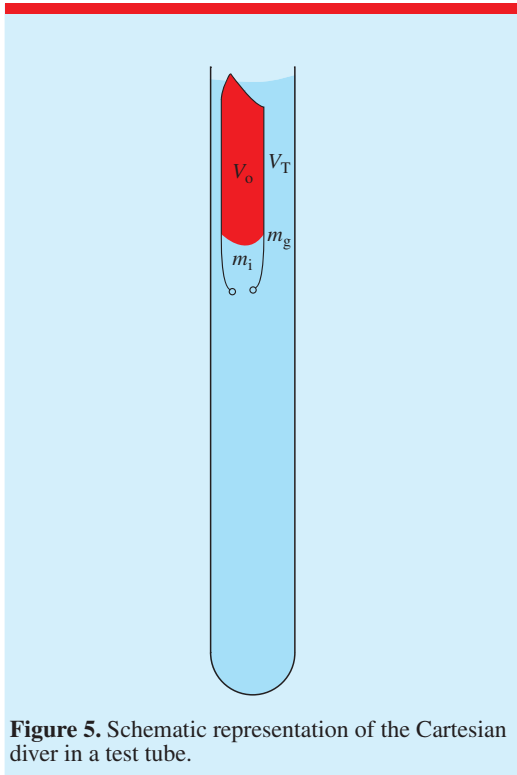


Figure 5. Schematic representation of the Cartesian diver in a test tube.

Therefore, an increase in pressure will correspond to a decrease in the volume of air in the diver, and thus the diver will sink, at least partially, in water. It is important to note that dp takes into account all variations in pressure. In this way, hydrostatic pressure is also considered during sinking.

We are now ready to explore the conditions for complete sinking of the diver. Assume that the initial and final thermodynamic states of air in the diver are characterized, respectively, by (the temperature being constant) (p_a, V_0) , $(p_a + \Delta p, V_0 - \Delta V)$, where Δp is a small pressure increase to which corresponds the volume decrease $-\Delta V$. Let us also set $V_{\text{ext}} \neq 0$ and $V'_{\text{ext}} = 0$ (the diver sinks after the variation of pressure Δp at its bottom is exerted). From (3) we can write, for smaller volume and pressure changes

$$\Delta V \approx \frac{\Delta p}{p_a} V_0. \quad (4)$$

Writing the equilibrium condition in the two states

$$\rho(V_0 - V_{\text{ext}}) = m_g, \quad (5a)$$

$$\rho(V_0 - \Delta V) = m_g, \quad (5b)$$

we have, by inspection, $\Delta V = V_{\text{ext}}$. Therefore, the pressure variation Δp_S necessary for sinking is given by (4) to be

$$\Delta p_S \approx \frac{V_{\text{ext}}}{V_0} p_a. \quad (6)$$

We conclude that, according to equation (6), for small values of V_{ext} , only a very small pressure change Δp_S is sufficient to make the diver completely sink. Furthermore, neglecting variations of hydrostatic pressure, we conclude that the expression for Δp_S given in (6) is approximately equal to the variation of the pressure exerted on top of the container.

In order to approximately calculate the critical height h_c below which the diver will irreversibly sink, we may still use equation (6). In fact, let us assume that, after having relieved pressure from the top of the test tube, the hydrostatic pressure in the bottom of the tube is the only additional pressure keeping the diver from re-emerging. In this way, we may set $\Delta p_S = \rho g h_c$ in (6), so that

$$h_c \approx \frac{V_{\text{ext}}}{\rho g V_0} p_a. \quad (7)$$

We note that, for $\frac{V_{\text{ext}}}{V_0} \approx 0.1$, the value of h_c is of the order of 1 m. One needs to use great care, therefore, to make this ratio small (adding or extracting water drop by drop with a syringe may help) in order to have irreversible sinking at heights of the order of some centimetres as shown in figure 4.

Simple analytic description of the observed phenomena: temperature change

Here we consider temperature variation as an externally perturbing quantity of the Cartesian diver state. A decrease in temperature of the water, from the initial room temperature T_{amb} to the refrigerator temperature T_f gives a decrease of water density ρ until about 4°C is reached and an increase of ρ down to 0°C . Air inside the diver may be considered to be at constant pressure (practically, atmospheric external pressure) while the temperature changes. In this way, the initial volume V_0 of air inside the diver, after cooling becomes V_f , where $V_f < V_0$. Because of the perfect gas law, neglecting variation in the hydrostatic pressure, we write

$$\frac{V_f}{V_0} = \frac{T_f}{T_{\text{amb}}}. \quad (8)$$

The absolute value ΔV of the air volume variation is thus given by

$$\Delta V = |V_f - V_0| = V_0 \left(1 - \frac{T_f}{T_{\text{amb}}} \right). \quad (9)$$

As before, we set $\Delta V = V_{\text{ext}}$ when we seek the condition for complete sinking. This gives the absolute value ΔT_S of the temperature decrease for complete sinking as

$$\Delta T_S = |T_f - T_{\text{amb}}| = T_{\text{amb}} \frac{V_{\text{ext}}}{V_0}. \quad (10)$$

Again, we conclude that, if $\Delta T > \Delta T_S$, the diver sinks. Using our diver in a test tube for non-negligible values of V_{ext} , we find that the diver sinks and the water becomes ice. Ask your students if, after thaw, the diver will rise. Some students will come to the conclusion that the diver will rise, having realized that the tube is too short to be able to observe irreversible phenomena.

Conclusions

Adopting a simple set-up for the Cartesian diver experiment, a rich variety of phenomenological aspects has been found. The apparatus can be easily realized, by an instructor, with a small test tube and eyedropper, using a little caution in using a gas flame to close the cut end of the latter glass element. In a school laboratory a ready-to-use system should be given to students.

One can show that many physical concepts are involved in this experiment. In fact, starting from hydrostatics, we are able, with the aid of a rather simple thermodynamic analysis, to illustrate the phenomenon of complete sinking of the diver under a small pressure increase Δp_S on the test tube. By using a long tube it is qualitatively argued that, for pressure variations higher than some critical value $\Delta p_c > \Delta p_S$, the diver will irreversibly sink. In particular, considering Δp_c as given solely by the hydrostatic pressure, the critical height h_c below which the diver will irreversibly sink is approximately found.

Repeating the same experiments considering temperature variation at constant pressure, we can show that similar conclusions can be reached. In particular, we note that complete sinking is obtained for the following simple expression of the temperature variation: $\Delta T_S = T_{\text{amb}} \frac{V_{\text{ext}}}{V_0}$. To be able to

place the container in a home refrigerator we need to use a test tube that is too short to detect irreversible sinking. Reversible sinking is found also when water is brought below freezing temperatures.

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References

- [1] Privat-Deschanel A 1869 *Traité Elementaire de Physique* (Paris: Hachette) pp 101–2
- [2] Frick J 1861 *Physical Technics* (Philadelphia, PA: J B Lippincot) p 93
- [3] Leybold Didactic Catalogue: Physics p 45 (www.ld-didactic.de)
- [4] Graham R M 1994 An extremely sensitive Cartesian diver *Phys. Teach.* **32** 182–3
- [5] Fakhruddin H 2003 Cartesian diver and riser *Phys. Teach.* **41** 53
- [6] Güémez J, Fiolhais C and Fiolhais M 2003 A demonstration apparatus for the Cartesian diver *Teach.* **41** 495–6
- [7] Wild R 1981 Ultimate Cartesian diver set *Am. J. Phys.* **49** 1185
- [8] Turner R C 1983 Toys in physics teaching: Cartesian diver *Am. J. Phys.* **51** 475–6
- [9] Orwig L P 1980 Cartesian diver ‘tricks’ *Am. J. Phys.* **48** 320
- [10] Miller J S 1954 Extension of the Cartesian diver experiment *Am. J. Phys.* **22** 235
- [11] Güémez J, Fiolhais C and Fiolhais M 2002 The Cartesian diver and the fold catastrophe *Am. J. Phys.* **70** 710–4
- [12] Siddons J C 1988 *Experiments in physics* (Oxford: Blackwell) chapter 3

Roberto De Luca obtained a degree in physics at the University of Salerno, Italy, in 1986, an MA in physics at the University of Southern California in 1987 and a PhD from the University of Naples and Salerno in 1992. Since 1994 he has carried out research at the University of Salerno. Besides research papers on Josephson junction array models, he has written more than 40 papers on physics teaching, some of which have been devoted to simple ways of introducing alternative energy concepts to young students.



Salvatore Ganci graduated in physics in 1972. He was professor of physics and mathematics at Liceo Scientifico ‘G Marconi’, Chiavari, Italy until 2004. His major interest is in the scalar theory of diffraction. Nowadays, as a freelance, he works in historical and pedagogical aspects of the physical sciences.

