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Čerenkov energy loss of muons in water

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Abstract. Using a large directional water Čerenkov counter and a near horizontal magnetic spectrograph, the Čerenkov energy loss of relativistic muons has been studied. The experimental results, which cover the range of muon momentum 0·3–120 GeV/*c*, are consistent with the classical theory of Frank and Tamm. They do not indicate a decrease in the Čerenkov loss, as expected according to Tsytovitch, nor do they confirm the large rise reported by Bassi *et al.*

1. Introduction

A suggestion that there is a reduction in the rate of loss of energy of charged particles traversing a dielectric medium at highly relativistic velocities has been made by Tsytovitch (1962 b, c). The results of Zhdanov *et al.* (1962) indicated the existence of such a reduction in the grain density of electron tracks in nuclear emulsion. The reduction in the measured *ionization* loss is the consequence of a suggested reduction in the Čerenkov contribution to the ionization loss at very high energies. According to Tsytovitch (1962 a), the corrections to the Čerenkov energy loss are only appreciable for particles having $\gamma \geqslant 50$ (γ being equal to E/Mc^2), and the magnitude of the reduction is of the order of 5–10% at the greatest energies ($\gamma \gtrsim 100$).

Experiments on the intensity of Čerenkov radiation have been carried out up to γ values of approximately 50 (Millar and Hincks 1957), but there are no data for the important higher values. The present paper reports the study of Čerenkov radiation up to γ values of 500, which is well above the value at which Tsytovitch considers the correction to be important.

2. Theory of Čerenkov energy loss

The theory of Čerenkov energy loss has been given by Frank and Tamm (1937), this being a classical treatment based upon Maxwell's equations. Although other theoretical studies have been made, notably those of Budini (1953), Ginsburg (1940) and Sokolov and Loskutov (1957), the basic theory of Frank and Tamm is still accepted as being correct and will be considered here as the basis for comparison with the experimental results. The Frank and Tamm theory leads to the following expression for the number of photons, of wavelength between λ_1 and λ_2, per centimetre of path generated by a particle of velocity βc moving in a medium of refractive index η:

$$\frac{\mathrm{d}N}{\mathrm{d}x} = 2\pi\alpha\left(1-\frac{1}{\beta^2\eta^2}\right)\left(\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right) \tag{1}$$

α being the fine structure constant. This expression has been evaluated for water with $\eta = 1{\cdot}368$ and the wavelength range 2500 Å to 4000 Å, and its form is shown in figure 4. On the basis of the Tsytovitch theory, a reduction in the intensity of Čerenkov loss for particles of ultra-relativistic energies is expected, the radiative corrections, with spatial dispersion neglected, being written in the form

$$\frac{W-W_0}{W_0} = \frac{e^2}{\pi\hbar c}\Delta_\mathrm{t}. \tag{2}$$

In the equation W_0 is the Čerenkov loss in the first approximation of the theory and Δ_t is

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the transverse part of the radiative corrections. For ultra-relativistic velocities Tsytovitch gives

$$\Delta_t = 2\ln^2\frac{E}{Mc^2}+2\left(\ln\frac{2E}{Mc^2}-1\right)\left(\ln\frac{M^2c^4}{E^2\chi_0{}^2}-1\right)+0{\cdot}1696 \tag{3}$$

for $E/Mc^2 \ll 1/\chi_0$, and

$$\Delta_t = \frac{7}{2}-\frac{\pi^2}{6}+\frac{1}{2}\ln^2\frac{1}{2{\cdot}72\chi_0{}^2} \tag{4}$$

for $E/Mc^2 \gg 1/\chi_0$, where

$$\chi_0 = \frac{e^2}{\hbar c}\frac{\omega_0}{\langle\omega_s\rangle}\ln\frac{c}{\langle v\rangle}.$$

The terms are as follows: $\langle\omega_s\rangle$ is the effective natural frequency of the atomic electrons, $\langle v\rangle$ their mean velocity and ω_0 is 2π times the plasma frequency.

For the purpose of the experiment described later $(W-W_0)/W_0$ has been evaluated for water in the following manner. For $c/\langle v\rangle$ it was assumed that electrons move in simple orbits according to Bohr's theory, and the mean value was found for the water molecule, namely 4·2/137. ω_s was taken as $2\pi I/h$, where I is the ionization potential of water, 67·5 ev. ω_0 was taken as $2\pi(Nc^2/\pi m)^{1/2}$, where N is the concentration of electrons of mass m in the medium. Thus for muons equation (3) is applicable for $E \ll 13{\cdot}7$ GeV and equation (4) for $E \gg 13{\cdot}7$ GeV. However, according to Tsytovitch, the radiative corrections are only appreciable for muons of energy greater than 10 GeV, in which case (4) shows that the correction for water for muons of energy very much greater than 10 GeV is a 3·8% decrease in the Čerenkov energy loss. Since the limiting energies are not known precisely it is not possible to give the shape of the curve or the point where the reduction begins. The magnitude of the reduction at the highest energies is shown in figure 4.

3. The experimental arrangement

3.1. *Introduction*

Near horizontal cosmic-ray muons, after traversing a magnetic spectrograph, passed through a large water-filled Čerenkov counter. The spectrograph incorporated neon flash tubes as trajectory defining elements and scintillation counters for triggering. It utilized an air-gap magnet, and the maximum detectable momentum of the instrument was 120 GeV/c. Whenever the spectrograph was triggered, the total pulse from the photomultipliers of the Čerenkov counter was displayed on a fast oscilloscope and recorded photographically. A large tray of flash tubes behind the Čerenkov counter made it possible to select only events attributable to single muons traversing the Čerenkov counter.

4.2. *The Čerenkov counter*

The counter used in the investigation consisted of a galvanized steel tank 180 cm × 120 cm × 120 cm containing water and four vertical light collectors. Each light collector comprised a 5 cm diameter quartz tube containing a liquid paraffin/POPOP mixture (5 mg POPOP per litre) viewed by a 53 AVP photomultiplier in optical contact with the liquid at the top of the tube. Particles traversing the counter entered and left it via the vertical 180 cm × 120 cm faces. The two inside 120 cm × 120 cm faces and the bottom of the tank were fitted with plane mirrors. Since the maximum angle of emission of Čerenkov radiation in water is 42°, the Čerenkov light striking the air–water boundary at the top surface of the counter was internally reflected into the water. The rear and front faces of the tank were painted black. The four light collectors in the tank were 1 m long and 36 cm apart. They were placed 7·5 cm in front of the rear black surface of the counter. At the bottom of each light collector was a small plane mirror.

In the ideal case a muon traversing the tank horizontally and centrally generates Čerenkov light in the water for a distance of nearly 120 cm. The light is given off in a cone of half angle 42° and thus all reaches the rear surface of the counter. However, for particles not central or horizontal, light from the earlier part of the particle's path in the water strikes the mirrors on the bottom and sides of the tank and perhaps the air–water surface and is reflected inwards towards the rear face of the counter. A small fraction of this light is incident on the light collectors and is absorbed by the POPOP, and then re-emitted isotropically and at longer wavelength than the incident light. A fraction of the re-emitted light travels along the light collectors and is detected by the photomultipliers. The light not incident on the light collectors is absorbed by the black surface of the counter.

An estimate of the number of photoelectrons produced at the four photomultipliers taken together, when a relativistic muon passes symmetrically through the tank, has been made assuming that the number of photons emitted is as given by (2), and that the absorption and emission characteristics of POPOP are as given by Swank (1958). The transmission properties of the water were considered and deduced from measurements made using an absorption spectrometer, and a cathode conversion efficiency for the photomultiplier of 10% was assumed. The value so obtained for the mean number of photoelectrons per muon was 18.

The pulses from the four light collectors were added together and displayed on an oscilloscope whenever a muon traversed the counter, as selected by the triggering scintillation counters of the spectrograph.

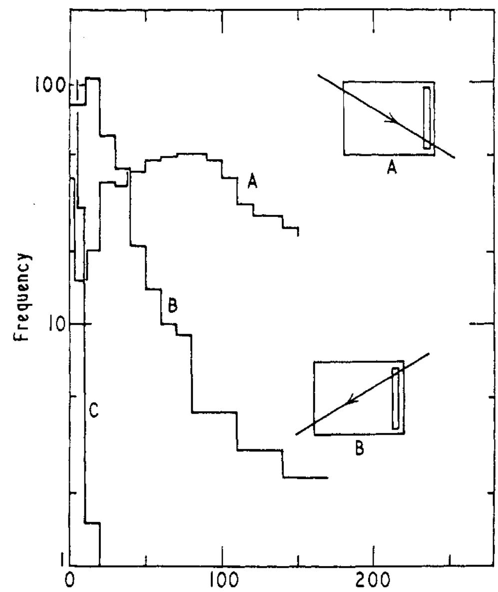

Figure 1. The distributions of the total pulse height from the Čerenkov counter for two directions of traversal of the counter by a muon. The directions are defined in the inset diagram and the noise distribution (C) is also shown. The curves are for equal numbers of triggers.

The performance of the counter was tested by studying its response to particles traversing the counter in opposite directions, as shown in the insets of figure 1. From the curves of figure 1 it is seen that muons traversing the counter in such a direction that their Čerenkov light falls on the light collectors directly give larger photomultiplier pulses than do muons going in the opposite direction. The curve for this latter class of muon indicates that a small fraction of the light produced in the tank does find its way to the light collectors. This is mainly due to incomplete absorption of the radiation by the black surfaces of the tank, but probably several other processes, such as knock-on electron production and photon scattering in the tank, contribute.

4. The results

Events were selected from those recorded in which only one particle traversed the spectrograph and was seen to leave the Čerenkov counter. Altogether 1765 events satisfied this criterion. The pulse-height distribution of these events is given in figure 2. The events were subdivided into those due to positive and negative muons, and the most probable pulse heights of the distributions are $(43{\cdot}9 \pm 0{\cdot}5)$ mv and $(43{\cdot}4 \pm 0{\cdot}9)$ mv respectively.

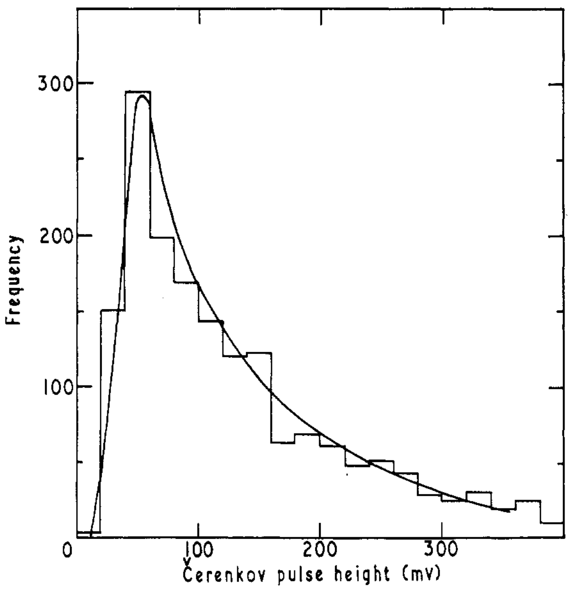

Figure 2. The pulse-height distribution for particles of all momenta.

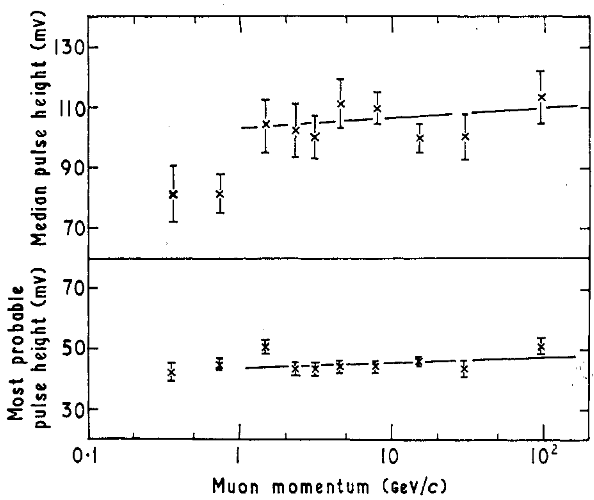

Figure 3. The median and mode of the Čerenkov pulse-height distributions as a function of muon momentum.

The most probable value of an asymmetric distribution of unknown form is a difficult quantity to estimate, and the method adopted in the present work was as follows. From the distribution of figure 2 a master curve was constructed, this being the average curve of those drawn by ten independent workers. Then the master curve was used to estimate the positions of the modes of the distributions corresponding to the various ranges of muon momentum. The position of the mode of each distribution was estimated by each of ten workers using the master curve and the mean value taken as the best estimate of the mode. The values so obtained are given in figure 3 where they are plotted against the mean momentum of the particles in each momentum cell. In this figure are plotted also the medians of the distributions. As an indication of the degree of consistency (or otherwise) of the Čerenkov loss in the relativistic region, the rate of rise per decade has been calculated for both the median and the mode of the distributions. The results are as follows:

$$M_{\text{median}} = (3{\cdot}6 \pm 2{\cdot}5)\% \text{ per decade of momentum}$$
$$M_{\text{mode}} = (4{\cdot}3 \pm 4{\cdot}3)\% \text{ per decade of momentum.}$$

In figure 4 the theoretical curve due to Frank and Tamm and the asymptotic limit according to Tsytovitch are shown, together with the observed most probable values, normalized to the theory at a momentum corresponding to the mean of all the points above 1 GeV/c.

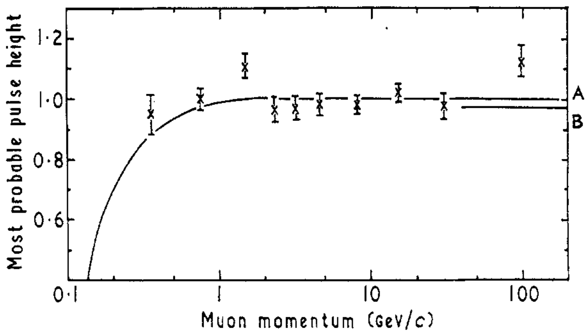

Figure 4. The most probable Cerenkov energy loss as a function of muon momentum. Curve A is from the classical theory and B is the limit of the reduction due to radiative corrections according to Tsytovitch (1962 a, b, c).

5. Discussion and conclusions

In the high-energy region only two experiments have been reported previously, namely those of Bassi *et al.* (1952) and Millar and Hincks (1957). The results of Bassi *et al.* cover the range of muon energy from 0·2 GeV to approximately 3 GeV and show a rise above that predicted by the classical theory of Frank and Tamm in the region of 3 GeV of about 26%. Millar and Hincks studying a similar energy range did not observe such a large rise, and their result is thus inconsistent with that of Bassi *et al.* They conclude that the theoretical Čerenkov intensity is given by the classical theory to within a few per cent from the threshold up to the plateau region approximately equal to 1 GeV/c.

The present work extends to much higher momentum than that hitherto, approximately 100 GeV/c, and the results expressed in figure 4 indicate that the Čerenkov radiation after attaining its plateau value at about 1 GeV/c remains comparatively constant up to the highest momentum studied. Thus the data are in good agreement with the predictions of conventional theory and with the observations of Millar and Hincks. There is no evidence from the results for a decrease of Čerenkov intensity above 10 GeV/c ($\gamma > 100$) as predicted by Tsytovitch; instead the last point, corresponding to muons of momentum greater than 45 GeV/c, shows if anything a small rise above the plateau value.

The data have been exhaustively examined from the point of view that positive and negative muons may give rise to opposite effects, so that the overall effect of Tsytovitch, which was calculated for negative electrons, is masked. However, no significant differences between the results for positive and negative particles have been found.

It is concluded therefore that the classical theory of Čerenkov radiation of Frank and Tamm up to muon energies of 100 GeV satisfactorily explains the observed phenomena and that there is no evidence for the existence of the Tsytovitch effect or for the big increase detected by Bassi *et al.*

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