Forward by Sol Garfunkel

While it is hard for me to believe, the Mathematical Contest in Modeling (MCM) is fast approaching its 30^{th} year. During this time we have grown from 90 US teams to over 5,000 teams representing 25 countries from all across the globe. We have been especially buoyed by the enthusiasm shown by our Chinese colleagues and the rapid growth in Chinese participation. COMAP welcomes your involvement with open arms.

COMAP runs three contests in mathematical modeling; they are MCM, ICM (the Interdisciplinary Contest in Modeling), and HiMCM (the High School Mathematical Contest in Modeling). The purpose of all of these contests has never been simply to reward student efforts — as important as that is. Rather, our objective from the beginning has been to increase the presence of applied mathematics and modeling in education systems at all levels worldwide. Modeling is an attempt to learn how the world works and the use of mathematics can help us produce better models. This is not a job for one country, but for all. The COMAP modeling contests were conceived and evolved to be strong instruments to help achieve this much larger goal.

It is my supreme hope that through this excellent book series the Chinese students will learn more about COMAP contests and more about the process of mathematical modeling. I hope that you will begin to work on the exciting and important problems you see here, and that you will join the MCM/ICM contests and the rewarding work of increasing the awareness of the importance of mathematical modeling.

Sol Garfunkel, PhD
Executive Director
COMAP
November 2012

Forward by Chris Arney

Undergraduate students who receive instruction and experiences in mathematical modeling become better and more creative problem solvers and graduate students. This book series is being published to prepare and educate students on the topics and concepts of mathematical modeling to help them establish a problem solving foundation for a successful career.

Mathematical modeling is both a process and a mindset or philosophy. As a process, students need instruction and experience in understanding and using the modeling process or framework. As part of their experience, they need to see various levels of sophistication and complexity, along with various types of mathematical structures (discrete, continuous, linear, nonlinear, deterministic, stochastic, geometric, and analytic). As a mindset, students need to see problems that are relevant, challenging, and interesting so they build a passion for the process and its utility in their lives. A major goal in modeling is for students to want to model problems and find their solutions. Recipes for structured or prescribed problem solving (canned algorithms and formulas) do exist in the real world, but mathematical modelers can do much more than execute recipes or formulas. Modelers are empowered to solve new, open, unsolved problems.

In order to build sufficient experience in modeling, student exposure must begin as early as possible – definitely by the early undergraduate years. Then the modeling process can be reinforced and used throughout their undergraduate program. Since modeling is interdisciplinary, students from all areas of undergraduate study benefit from this experience.

The articles and chapters in this series expose the readers to model construction, model analysis, and modeling as a research tool. All these areas are important and build the students' modeling skills. Modeling is a challenging and advanced skill, but one that is empowering and important in student development. In today's world, models are often complex and require sophisticated computation or simulation to provide solutions or insights into model behavior. Now is an exciting time to be a skilled modeler since methodology to provide visualization and find solutions are more prevalent and more powerful than ever before.

I wish the students well in their adventure into modeling and I likewise wish faculty well as they use the examples and techniques in this book series to teach the modeling process to their students. My advice to all levels of modelers is to build your confidence and skills and use your talents to solve society's most challenging and important problems. Good luck in modeling!

Chris Arney, PhD
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Professor of Mathematics
Director of the Interdisciplinary Contest in Modeling
October, 2011

Chapter 1 Introduction

The Mathematical Contest in Modeling (MCM) is an annual international mathematical event, where teams of undergraduate students from different countries work out mathematical models around the clock to solve open-ended problems. These are thought-provoking problems with real sense of applications. Their solutions are presented in the form of research papers, and the contest is as much a contest in clear writing as it is in mathematical modeling. In some ways, clear writing is the more important part of the contest: A well-written paper using a poor model will typically do better than a poorly written paper using a good model.

1 A brief history

The MCM contest was originally conceived of as an "Applied Putnam" contest. The Putnam Competition, started in 1938, is an annual American mathematical competition where students are given problems to work on individually. The scores on the Putnam tend to be very low: Out of a possible 120 points, many students get zero and about half get two or fewer points.

In the early 1980's, Ben Fusaro, who founded the MCM, was concerned about the lack of applied problems on the Putnam and realized that students find the low scores discouraging. He thought that a good balance to the Putnam would be a contest where teams of students would work on applied math problems. For example, students would work on two problems in one day, one problem involving discrete math *and* the other involving continuous math. This idea received strong support from his colleagues, with a new format where the teams work on one problem over a weekend, either a problem involving discrete mathematics *or* a problem involving continuous mathematics. Solomon Garfunkel, the director of the Consortium for Mathematics and Its Applications (COMAP) suggested that Ben Fusaro submit a proposal to Fund for the Improvement of Postsecondary Education (FIPSE), a program of the U.S. Department of Education. A three year grant was approved in June 1984, which provided financial support to start the first MCM contest in next year. The final proposal to FISPE describes the purpose of the MCM contest [1]:

> The purpose of this competition is to involve students and faculty in clarifying, analyzing, and proposing solutions to open-ended problems. We propose a structure which will encourage widespread participation and emphasize the entire modeling process. Major features include:
>
> - The selection of realistic open-ended problems chosen with the advice of working mathematicians in industry and government.
> - An extended period for teams to prepare solution papers within clearly defined format.
> - The ability of participants to draw on outside resources including computers and texts.
> - An emphasis on clarity of exposition in determining final awards with the best papers published in professional mathematics journals.
>
> As the contest becomes established in the mathematics community, new courses, workshops, and seminars will be developed to help students and faculty gain increased experience with mathematical modeling.

The first contest, in 1985, was merely a national contest with only 158 US teams entered and no international teams. The contest grew quickly, largely due to international support, which was somewhat unexpected when the idea of the MCM contest was conceived. By 2010 there were 2,254 teams entered, most of them were international, and the number of US teams was also increased to 358.

The MCM contest is organized and administered by COMAP. In each year, the contest consists of two problems, labeled as Problem A and Problem B. Problem A deals with modeling using continuous math, while Problem B deals with modeling using discrete math. COMAP added the Interdisciplinary Contest in Modeling (ICM) in 2000, which takes place at the same time as the MCM contest and follows the same rules. The ICM contest in each year consists of only one problem, labeled as Problem C, which deals with modeling using continuous math, discrete math, or both. Thus, the American mathematical contest in modeling is also referred to as the COMAP MCM/ICM contest. For simplicity, we will use the MCM to refer to both the MCM and the ICM contests, unless otherwise specified. COMAP also organizes the High School Mathematical Contest in Modeling (HiMCM), which is specifically designed for high school students and will not be discussed in this book.

A team taking part in the MCM contest can only work on the problem, and hence the paper, once the contest starts. However, the team should prepare before the contest starts. The team itself should have been chosen carefully. Ben Fusaro once said "It is not the three best that make the team, it is the best three." In other words, the emphasis should be on having a team whose members work well together and whose skills complement each other. As well as a good math student, a team should have someone who is familiar with available computer programs and someone who can write programs. One person can of course fill more than one role.

2 Classifications and judging

MCM contest problems do not have standard solutions, and there are no passing grades. Papers are classified into categories, rather than receiving numerical scores.

Judging criteria

For each MCM paper, the judges will look for the entire modeling process and the clarity of its exposition. Your paper will be judged based on the following criteria:

- Does your paper provide a satisfactory interpretation to the contest problem, with necessary clarifications to vague concepts (if any)?
- Does your paper list all the assumptions used in your modeling, and provide satisfactory explanations or justifications to these assumptions?
- Does your paper provide sufficient analysis that motivates or justifies your modeling approach?
- Does your paper design a model (or a series of models) that can effectively solved the contest problem?
- Does your paper test the stability of the model you constructed?
- Does your paper discuss the strengths and weaknesses of your model and state clearly what results are obtained?
- Does your paper provide a satisfactory summary?

Papers with incomplete solutions are acceptable, and may even obtain a good evaluation if it provides a unique and creative approach to the problem.

Classifications and awards

Papers that do not sufficiently address the MCM problem or somehow violate the contest rules are labeled **Unsuccessful Participant**. The rest of the papers are classified, based on the judging criteria, into five categories. They are **Successful**

Participant, **Honorable Mention**, **Meritorious**, **Finalist**, and **Outstanding Winner**. Any paper that is reasonably well written, addresses the modeling problem, and meets the contest requirements is a Successful Participant. Outstanding Winner papers are the best written papers that present the best models. The categories and the corresponding paper percentages are listed below:

- Successful Participant. About 50% of the papers.
- Honorable Mention. About 30% of the papers.
- Meritorious. Usually 10% to 15% of the papers.
- Finalist. About 1% of the papers.
- Outstanding Winner. About 1% of the papers.

In addition to paper classifications, MCM papers may also receive the following awards:

- The INFORMS Award, created by the Institute for Operations Research and the Management Sciences (INFORMS), may be given to a selected Outstanding Winner paper from solutions to each of the three problems.
- The SIAM Award, created by by the Society for Industrial and Applied Mathematics (SIAM), may be given to a selected Outstanding Winner paper from solutions to each of the three problems.
- The MAA Award, created by the Mathematical Association of America (MAA), is given to a selected Outstanding Winner paper from, respectively, solutions to Problem A and Problem B.
- The Ben Fusaro Award, created by COMAP, is typically given to a finalist paper that is especially creative but contains a flaw that prevents it from attaining the Outstanding Winner designation.

Judging process

The judging occurs three weeks after the contest is over and takes place over a weekend. The judges do not know who wrote the papers; the papers are identified by control numbers assigned by COMAP and the names and institutions of the contestants are removed from the papers before the judges see them.

The judging of the papers occurs in two stages. The first stage of judging is **triage**, where the papers are divided into three groups: (1) papers which are good enough to go on to the next round, (2) papers which are good but not good enough for the next round (these are classified as Successful Participants), and (3) papers which do not meet the standards of the competition (these are labeled Unsuccessful Participants). For this stage, there is a head judge and about one additional judge for each twenty-five papers. Each paper is read by two judges, each of whom gives the paper a score. If the two scores are not very close, the two judges discuss it and, if they cannot agree on the quality of the paper, a third judge is conferred. Once two reasonably close scores are given to a paper, the scores are added to give the paper an overall triage score. The head judge and the contest director determine a cutoff score for the papers. Less than but close to half of the papers will have scores above the cutoff and go to the next round.

The triage judges spend about ten minutes looking over each of their papers, and because of the time limitation they mainly look at the summary to determine the quality of the paper. **This makes the summary the most important part of the paper.**

For the post-triage judging in the second stage, the continuous and the discrete math papers are separated and each have their own sets of judges. There will be several rounds, and papers which make it past one round are read by different judges for the next round. The more rounds a paper passes through, the higher the classification of the paper will be. The judges will spend between fifteen and thirty minutes reading each paper. To impress the judges in so little time, the papers need to be well organized and highlight the important results.

In the final round, where the Outstanding Winners are selected, the papers are read and discussed by all the judges. To be an Outstanding Winner, all of the judges, as well as the contest Director and Associate Director, have to agree that the paper deserves it.

3 Writing is important

Given the importance of writing for the contest, more than one person has suggested that the team include an English major (or two!). At the very least, a team should include very good writers. For a month before the contest the team should meet a few times a week to prepare. The team advisor can assign practice problems for the team to work on; the team members should practice researching the problems, solving them, and writing up solutions. Since the contest will not have started yet, the writing can be critiqued and this is a good time to filter out bad habits. Probably the best way to become a good writer is to read good writing, and so the best way to prepare for writing up an MCM paper is to read previous winners. Each year the Outstanding Winners, along with commentaries, are published in the *UMAP Journal*.

When the contest begins, it is a good idea for each team member to initially take on specific duties. For example, Alice should do any necessary background research on the problem, Bob should begin writing the paper, and Charlie should do any necessary coding.

The writing should begin as soon as the contest starts, for many students tend to underestimate how long it will take to write the paper. Sometime during the second day of the contest the team may want to consider finishing the work on the model and concentrate on the writing.

Chapter 2 Paper Structure

The paper that you submit to the MCM must be designed so that it can be read and understood quickly. For this purpose you must first come up with a good structure for your paper. A poorly organized paper can hide good results, so you need to make sure that your paper is logically organized and carefully laid out. Because the contest judges have little time to spend on your paper each round, your paper should catch the judge's interest and highlight the important points.

1 Good structure goes a long way

Your paper should be organized in sections and subsections, and they should be titled in such a way that merely reading them will provide an outline of the paper. Listed below is the suggested outline for an MCM paper [2] based on the judging criteria:

- A clarification or restatement of the problem, as appropriate.
- A clear exposition of all assumptions and hypotheses.
- An analysis of the problem justifying or motivating the modeling to be used.
- The design of the model.
- A discussion of how the model can be tested, including error analysis and stability (conditioning, sensitivity, etc.).
- A discussion of the strengths and weaknesses of the model.
- A one-page summary of the results, typed on the Summary Sheet, must be attached to the front of each copy of the solution paper.

A textbook example is the paper "How to Please Most of the People Most of the Time" [3] from the 1989 MCM, discussing the best way to line up airplanes for takeoff, to minimize dissatisfactions of passengers because of delays. The followings are the sections and subsections of this paper:

Summary

1 *Restatement of the Problem*

2 *Assumptions*

3 *Justification of Our Approach*

4 *The Model*

  4.1 *Dissatisfaction of a passenger needing a connection*

  4.2 *Dissatisfaction of a passenger not needing a connection*

  4.3 *Total dissatisfaction on an aircraft*

5 *Testing the Model*

6 *Results*

7 *Strengths and Weaknesses*

  References

While not every paper will have sections so clearly lined up with the suggested outline, if you give your paper a simple and standard sectioning it will make it easier for judges to get through the paper quickly and with great comprehension.

When breaking up your paper into sections, you should avoid having large sections of homogeneous text, since it is easy for the reader to gloss over them when skimming a paper. You should highlight important sentences by putting them in bold face. Newly defined words should also be bold. Note that you may also highlight important sentences or words in italics, but bolding them will make them standout better for someone skimming the paper. Overusing bold or italics, however, will make your paper look awkward and so you should keep the use of them to the minimum.

Important equations should be displayed rather than inline. A list of items, such as a list of hypotheses that you will need to make to construct your model, should be presented as a bulleted list rather than a paragraph. When reasonable, information should be given as a figure or table with a short, informative caption.

2 Writing the introduction

The introduction to your paper should be the first part of the paper you write, even though it may not have the title "Introduction" (for example, you may use "Restatement of the Problem"). Your introduction should include your interpretation of the contest problem, your survey and discussions of the existing work related to the problem, and your thoughts and approaches to solving the problem.

The introduction is where you need to show the reader that you have put significant thought into the problem. You should begin writing the introduction as soon as you begin working on the problem, and you should revise it a few times during the course of developing your solution.

The first sentence of your introduction is the most important sentence of your paper and should convince the reader that they want to read the rest. The entire first paragraph should be accessible even to people unfamiliar with the problem at hand and get them interested. This paragraph should not be mathematically heavy and symbols should be kept to a minimum. Thus, composing the first sentence and the first paragraph may often require substantial thought and perhaps a number of revisions.

Once you have decided which contest problem to work on, the first thing you should do is to restate the problem in your own words. You need to make your goal precise and possibly clarify anything ambiguous about the original problem statement. Many problems are subject to interpretation, so you need to let the reader know how you see the problem. For example, in the 2010 contest one of the problems was to explain the sweet spot on a baseball bat. There is more than one meaning of what a "sweet spot" is, and so in the introduction to "The Sweet Spot: A Wave Model of Baseball Bats" [4] (one of the Outstanding Winners), the authors specify:

> *There are at least two notions of where the sweet spot should be – an impact location on the bat that either*
> - *minimizes the discomfort to the hands, or*
> - *maximizes the outgoing velocity of the ball.*
>
> *We focus exclusively on the second definition.*

Even apparently precise information may still benefit from a nice interpretation. For example, one of the problems from 1988 was to develop a search method to find a fleeing boat given some information about the boat's current position. The information included angles "accurate to within 2°". One of the Outstanding Winners, titled "Locating a Drug Runner: Miami Vice[①] Style" [5], stated how they interpreted the accuracy:

> *We interpret the error of* $\pm 2^\circ$ *as a normal distribution ... with standard deviation of* 1°.

Once you know exactly what you are working on, you should begin to search the literature for published works on similar problems and the models that were

① "Miami Vice" was an American television crime drama series aired from 1984 to 1989, where two police detectives worked undercover in Miami.

used. Your introduction should include your restatement of the problem and a discussion of previous work on similar problems. Your introduction should also include a brief description of your approaches to the problem and your major results.

3 The body of the paper

After the introduction, your paper should begin with a construction of the model (or series of models) that you are going to use. This should include your assumptions, your explanations why these assumptions are reasonable, and your descriptions of your model.

To begin, you need to state very clearly what assumptions you are using to create your model. There should be no unstated assumptions, the reader should never have to wonder what you are doing or why you are doing it.

This section illustrates, using examples from outstanding winners, how to provide explanation to assumptions, how to build a nice model from a series of simpler models, and how to build a general model that contains the contest problem as a special case.

Assumptions and justifications

One of the problems in the 2010 contest was to develop a method to predict the location of a serial criminal based on the location of the crimes. The motions of the criminal needed to be considered. One of the Outstanding Winners, "Centroids, Clusters, and Crime: Anchoring the Geographic Profiles of Serial Criminals" [6], modeled the region of interest as a certain type of metric space. To justify this, the authors, in a section conveniently titled "Assumptions", gave a bulleted list of what they wanted to assume and why they could assume it. One of their assumptions was "Criminal's movements are unconstrained". The movements of a person in a city would normally lie on a grid and be constrained by buildings and walls, so the authors follow their assumption with an explanation:

> ***Criminal's movement is unconstrained.*** *Because of the difficulty of finding real-world distance data, we invoke the 'Manhattan assumption': There are enough streets and sidewalks in a sufficiently grid-like pattern that movements along real-world movement routes is the same as 'straight-line' movement in a space discretized into city blocks ...*

A lesser paper would have left the issue open, and perhaps simply used unrestricted motion of the criminal as an unstated assumption. Always explicitly state any assumptions that you use and provide justification for your assumptions; if you cannot, you should rethink your assumptions. The judges will look at your assumptions, as well as how reasonable and how justified they are.

Model designs

When creating the actual model, keep in mind that the model is a tool used to get your results and not an end in itself. The model itself does not have to be elaborate and in many cases should not be. The important thing is the information you get from the model. As with most things, **the simplest model that does the job is probably the best.**

During your background research for the problem you will probably come across models that you could modify for the current problem. If possible, you should build upon previous results. The model you use should be something that you could have come up with yourself (otherwise it will probably be too complicated).

Model series

One approach to model building is to create a series of models, starting with something simple and making it more realistic along the way. For example, a problem from the 2008 contest was to predict the effects of an increase in global temperatures (and the resulting melting of the North Polar ice cap) on the coast of Florida. One of the Outstanding Winners, "The Impending Effects of North Polar Ice Cap Melt" [7], produced such a series of models as follows:

The authors of the paper first considered **Model 1: Constant Temperature**, where the global temperatures did not increase or decrease and there was no change in the rate of the ice melting and rate of increase in the ocean volume. They then considered **Model 2: Variable-Temperature Model**, where they take into account a changing global temperature. Finally, they get **Model 3: Ocean Volume under Warming**, where they take into account the previously ignored relative surface areas of the Northern and Southern hemisphere oceans (a consideration that was not brought up in the problem statement).

When you create your model, you should focus on a single model (or, as above, a series leading to a single model). Some teams present their attempts of creating several superficial models and hope to score a few points from them. This detracts from the paper and is viewed unfavorably by the judges. On the other hand, you should also try to avoid designing everything from scratch, for it is hard to design a good model from scratch in a short time.

Modeling for general problems

Another approach that you should avoid is creating a model too specifically designed for the problem. While your problem should ultimately be answered, the better papers view the problem as an instance of a more general problem and deal with the larger problem before focusing on the specific problem. As an example, one problem from the 1988 contest was to load two railroad flatcars, each of a given length and capable of holding a certain weight, with as little wasted space as possible. The widths and weights of the crates to be loaded were given, as well as the number of each type of crate. There was clearly a more general problem — you could leave the specific values given in the problem unspecified — but the specific values of the widths and weights in the problem could be used to greatly simplify the problem. A lesser paper would only examine the simpler, specific problem; but one of the Outstanding Winners, "Loading Two Railroad Cars" [8], wrote:

> *We have produced a general algorithm to solve this type of problem, but for our problem a relationship exists that greatly simplified the algorithm.*

They then proceeded to solve the specific MCM problem.

4 Writing a conclusion

You should conclude your paper with a description of model testing and your results. Any models you build are a means to an end. The goal is to answer the question being asked. Even though the problems are open-ended, after creating and analyzing your model (or models) you should reach a definite conclusion. The answer should be clearly stated at the beginning of the section containing your conclusion (and this section is often conveniently titled "Conclusions"); the reader should not have to search for your answer or work at interpreting it. You should discuss your answer, but make it clear what part of the discussion is your opinion and what part is your conclusion.

You are asked to check the sensitivity and stability of your model. This is a stated requirement, meaning that doing it will make your paper look good. Unfortunately, many teams do not put much effort into this part. If your model requires some parameters (for example, a traffic problem might depend on the average speed of the traffic), you should see what happens to your model and solution when the values of the parameters are slightly altered.

You also need to provide evidence to support your solution as a good, if not the best, answer. Depending on the type of problem being asked, your solution might be a procedure of some sort. In this case, you would want to test it multiple times, using slightly different input each time. When possible, use real data to avoid the suspicion of contrived data.

When you write up your results, be sure to include sufficient information for the reader to be able to recreate everything necessary; if part of your result involves the output of a computer program, be sure the reader knows what program was used and present the algorithm you use for the part of the coding your team wrote.

Given the open-ended nature of the MCM problems and the limited time that you have to work on it, your model and solution will almost certainly have deficiencies. The judges will certainly notice problems with your work; you need to show them that you are astute enough to see them yourselves. While the limitations of your model may have been mentioned in your model description, you should still mention them again when you discuss the strengths and weaknesses of your model.

Another consequence of the limited resources you have while working on your problem is that there are many things you were unable to do. Your paper should mention things you could do should more time and resources be available.

5 Writing the summary

The most important part of an MCM paper is the summary sheet. Each paper is required to have a summary sheet attached to the front, and this will basically determine whether or not your paper makes it past the first evaluation stage (triage). A good model and a good solution will not make it past triage if it does not have a good summary.

According to the contest rules [9], the summary sheet should include

- **Restatement and clarification of the problem:** State in your own words what you are going to do.
- **Explain assumptions and rationale/justification:** Emphasize the assumptions that bear on the problem. Clearly list all variables used in your model.
- **Include your model design and justification** for type model

used or developed.

- **Describe model testing and sensitivity analysis**, including error analysis, etc.
- **Discuss the strengths and weaknesses** of your model or approach.

The summary should only be a little over a half page, summarizing your paper, but it should be written from scratch with considerable efforts and not simply be snippets from the paper. Thus, composing a good summary takes tremendous effort and you should revise it a few times until you are satisfied with it.

The summary should get the reader interested in the paper and want to know the details. Make sure it is not a dry recitation of what's in the paper ("First we ... then we ...") and you should avoid a beginning of the sort "This paper will solve ...". In particular, you should come up with a attractive first sentence to grip the reader's attention. For example, in 2008, one of the problems was to create Sudoku puzzles. The summary of one of the Outstanding Winners, "Taking the Mystery Out of Sudoku Difficulty: An Oracular Model" [10], begins

> *In the last few years, the 9-by-9 puzzle grid known as Sudoku has gone from being a popular Japanese puzzle to a global craze.*

While the summary cannot, and should not, provide too many details, it should still clearly describe your approach to the problem. It should include all of the main points and big ideas, and be specific about the conclusion. It should include numerical results when appropriate, but equations should be avoided if possible.

Chapter 3 Style

You would want anyone reading your paper to notice your mathematical work and your model, not your writing. Thus, your writing should be as unobtrusive as possible. That does not mean it should not be dynamic. The reader should be smoothly led through your work, just without being distracted by awkward phrasing or stilted sentences.

While reading your paper, the reader should be thinking about the problem and your work on it. Any time that the reader needs to pause, such as for parsing your sentences or for recalling the facts brought up in the paper that had not been used for a while, is time taken away from thinking about your solution. Given the little time that the judges have to read your paper, this can prevent your paper from advancing.

MCM papers are math papers, and math papers have certain conventional styles. Judges are used to these styles and expect that your papers are written in such styles. Thus, learning to write in these styles can help you write a better paper. This chapter contains some suggestions of what to do and examples of mistakes to watch out for.

1 Write in the first person, plural

The papers submitted to the MCM, and math papers in general, are typically written in the first person plural; throughout the paper "We" see what is going on and "We" draw conclusions. The number of authors is irrelevant; the "we" does not refer to you and your coauthors but rather to you along with the reader. You are leading the reader through the paper. First person plural is certainly more friendly, which helps shorten the distance between you and your reader. Moreover, when describing things, using "we" as the subject of a sentence is very convenient. In the unlikely event that you need to refer to yourself specifically, you can write "the authors" or "one of the authors".

The convention of using the first person plural in mathematical writings may differ from what you are used to see from mathematics textbooks written in Chinese, and so you should pay special attention to not using the third or the second person in your MCM paper.

2 Use the present tense

Whatever is being done at the moment can reasonably be referred to as the present: "We [now] solve the equation.". The previous parts of the paper would then be the past: "We solved the equation in the previous section." The unread parts of the paper would naturally be the future: "We will solve the equation in the next section".

In other words, if we view writing a paper as writing an electronic file by moving a cursor from the beginning to the end, then while you're writing, to describe what happens at the current location of the cursor is the present, what happened before the cursor is the past, and what will happen after the cursor is the future. While using the present perfect may sometimes make sense for things occurred or done before the current location of the cursor, using the simple past is every bit simpler and cleaner in mathematical writings.

When surveying other people's work, what they did is the past, but the theorems and facts are the present. For example,

They showed that the problem is NP-complete.

3 Use the active voice

When describing the same thing or stating the same fact, one can use the active voice or the passive voice, but active voice is more direct and more powerful. Math has strong logic, but may also be dry, and so using active voice can make math contents clearer and more dynamic.

Recall, the active voice is where the subject of the sentence performs an action:

The model produced a desirable conclusion.

Here, the model does the producing.

The passive voice is where an action is performed on the subject:

A desirable conclusion was produced.

or

A desirable conclusion was produced by the model.

In these examples, the desirable conclusion is the result of the production, not the cause.

Chinese students tend to write math papers in the passive voice. This phenomenon may be an outcome of their early training in technical writings where the passive voice is often used. Whenever you have doubts, please remember: The active voice is more concise and more dynamic than the passive voice. You should write in the active voice whenever possible.

4 Use simple sentences

The reader of your paper should not have to slow down to parse your sentences or guess what you want to say. You should use sentences which are as simple as possible, such that one sentence only describes one idea. The verb should be close to the subject. Complicated sentences can often be broken into simpler sentences. For example, the sentence

Bad: The value of the parameter λ, which was used in the previous section to determine the height of the building, can also be used to determine its width.

can be rewritten

Good: In the previous section, we used the value of the parameter λ to determine the height of the building. We can also use λ to determine the width of the building.

5 Use short paragraphs

A paragraph is used to represent a bigger idea and describe a larger event, often consisting multiple sentences. The end of a paragraph gives the reader a chance to pause and think about the paragraph. If you use short paragraphs, it will be easier on the reader. On the other hand, you should also avoid extremely short paragraphs of one or two simple sentences. Too many extremely short paragraphs would cause the reader to pause too often, which would distract the reading flow.

6 Use words which are as specific as you can

You should avoid vague or abstract words as much as feasible; concrete words are easier on the reader. For example, instead of saying

Bad: Using the previous expression, we can conclude ...

you should probably say

Good: Using the previous equation, we can conclude ...

7 Do not include trivialities

Your paper should include enough details for the reader to follow what you are doing, but if you include too many trivial details then the reader could become bored or annoyed with your paper. Any routine computations, for example, should probably be omitted.

Exactly what constitutes triviality is a judgement call. Whenever you have doubts, please remember: Reviewers of your paper are experts. They want to learn your flow of thoughts as quickly as possible without being bogged down with trivialities.

8 Highlight important statements

You should make important statements stand out so that they will draw the attention of your readers. A single important statement should probably be put in bold font. Putting it at the beginning of a paragraph which explains the statement would probably help the reader the most. A word being defined should also be put in bold font.

If you have several related important statements, such as the assumptions you are using to create your model, they can be highlighted by putting them in a bulleted list. In many outstanding MCM papers, for example, the section "Assumptions" only consists of such a bulleted list.

9 Omit needless words

Many all too common constructions are needlessly complicated. For example,

Bad: The variable x is positive, due to the fact that it is a perfect square.

should be simply

Good: The variable x is positive since it is a perfect square.

As an another example, the following passage is extracted from the abstract of a paper; the abstract does not explain what it means by "working and intermediate nodes":

Bad: In this paper, taking energy consumption into consideration, we rethink the virtual-network embedding problem through (1) the rebuild of the network model by considering the power consumption of both the working and intermediate nodes; and (2) the design of an energy-aware heuristic and optimized virtual-network embedding algorithms under this new network and power model.

It can be rewritten as

Good: We present an energy consumption model for the virtual-network embedding problem and devise an energy-aware virtual-network embedding algorithm to reduce power consumption.

Whenever you see several words taking the place of a single word, you should use the single word instead. Many phrases using a noun ending in "ion" can be shortened. For example, instead of writing

Bad: We will now find the solutions of the following equation.

you should write

Good: We will now solve the following equation.

You will need to be careful of shortening a sentence too much, of course. While the sentence

Bad: We will assume $x > 0$.

may seem like a suitably shortened version of

Good: We will assume that $x > 0$.

the "that" in this case helps the reader through the sentence. In general, if mathematical expressions are followed, the words "assume" and "suppose" should be followed by "that".

10 Use parallel phrasing to emphasize similarities

Similar phrasing should be used when there are similarities between objects; it makes the reading easier and emphasizes connections. For example, instead of writing

Bad: If a *is a root of the function* f*, then the function* g *has root* b.

you should write

Good: If a *is a root of the function* f*, then* b *is a root of the function* g.

or

Good: If the function f *has a root* a*, then the function* g *has a root* b.

11 Avoid pointless repetition

While similar phrasing should be used to emphasize similar objects, it otherwise should not be overused. Monotonous sentence structures will make your writing boring. For example, instead of writing

Bad: First we solved an equation. Then we solved another equation.

you should change the second sentence so it is distinct from the first. If appropriate, something like

Good: First we solved an equation. The solution of the first equation allowed us to solve the second equation.

would be an improvement.

Simply changing key words is not enough to avoid boring repetition. A paragraph that looks like

Bad: We know ... Hence, ... It follows that ....

will make the reader lose interest quickly. For example, the following sentences

Bad: We know that Equation A has a solution. Hence, Equation B has a solution. It follows that Equation C has a solution.

would probably bore or even annoy the reader. The following is an improvement:

Good: We know that Equation A has a solution, which implies that Equation B has a solution. Thus, Equation C also has a solution.

12 Avoid using the same word for different purposes

If you use the same word (or similar words) multiple times in different ways, it will look odd to the reader and they will need to pause to consider it. For example, instead of

Bad: We ran into several problems while trying to solve the problem.

you should write

Good: We ran into several obstacles while trying to solve the problem.

You should make sure you have a thesaurus handy to be able to come up with synonyms as appropriate.

13 Keep the antecedents of any pronouns clear

Any pronoun that you use refers to a previous noun. While a pronoun typically refers to the last noun before the pronoun, you should make sure that it is clear which noun is being referred to. For example, in the sentence

Good: The function is continuous; it is also nonnegative.

the pronoun "it" refers to the function. However, in the sentence

Bad: The function and its derivative are continuous; it is also nonnegative.

the first "it" (in "its") refers to the function, but it is not clear what the second "it" refers to; does it refer to the function or the derivative? In this case, the second pronoun should be avoided and the sentence should be rewritten. For example, if the second "it" refers to the function, then you may want to rewrite the sentence to something like

Good: The function and its derivative are continuous; the function is also nonnegative.

Likewise, if the second "it" refers to the derivative, then you may want to rewrite the sentence to

Good: The function and its derivative are continuous; the derivative is also nonnegative.

Whenever you write "it" or "this", be sure it is obvious what the "it" or "this" refers to.

14 Do not overstate your results

While you want to emphasize important results, you should not need to build them up too much. By inflating their importance, you open the possibility of disappointing the reader. Once the reader has a statement pointed out to them, you should leave it to them to judge how important it is. In particular, exclamation marks should be avoided.

Chapter 4 English Usage

You should be careful to use proper English throughout your paper. Even small mistakes will be noticed by some of the readers and, whether or not they regard it as important, once it is noticed it will be a speedbump. The more speedbumps there are, the longer it will take to read the paper and the less pleasant it will be.

Natural languages are organic, and the difference between a hard and fast rule and a suggestion is never clear. For just about any rule, you can probably find a good author who breaks that rule. However, the MCM contest is not the time to experiment with the language. Ignoring the standard rules will be noticed and it is possible that you will be violating a rule near and dear to the heart of one of the judges.

You should have a book on proper English usage, such as Fowler's *A Dictionary of Modern English Usage* [11], available, and consult it frequently. The rest of this chapter is devoted to mentioning some specific things to watch out for.

1 Make sure your subject and verb agree

If the subject and verb of a sentence disagree (both must be singular or both must be plural) it will stand out to the reader. Be sure to double check agreement whenever there is any doubt.

Making sure the subject and verb agree is usually straightforward when the subject is simple and close to the verb, although there are some cases which can be tricky. Ensuring agreement is less simple when there is a phrase between the subject and verb. In a sentence like

One of the solutions is positive.

even though several solutions are implied, the subject is the singular "One", and so gets a singular verb. In the sentences

The positive solution and the negative solutions are ...

The positive solution or the negative solutions are ...

the subjects are plural, however in the sentence

The positive solution, as well as the negative solutions, is ...

the subject ("The positive solution") is singular.

2 Use "that" and "which" correctly

While the importance of properly distinguishing between "that" and "which" depends on the reader, it is a topic in many books on good style and it is worth your trouble to get it right.

The word "that" is used to specify an object; in the sentence

The car that was blue went through the stop sign.

there may be several cars, but the phrase "that was blue" specifies the car under discussion. Without the phrase, the reader may not know what car went through the stop sign.

The word "which" is used to add information about an object; in the sentence

The car, which was blue, went through the stop sign.

the reader already knows what car is being discussed. The writer is adding information about the color of the car.

As rule of thumb, when deciding whether to use "that" or "which", use "that" whenever it sounds good.

3 Use correct spelling

Be sure to use a spellchecker on your paper. Since the MCM contest is based in the United States, you should have your spellchecker set to check for American English.

A spellchecker can only check if a word is spelled correctly, and it does not check to make sure you are using the correct word. If there is any doubt about the meaning of a word you are using be sure to look it up. Here are some words which are spelled similarly but have different meanings.

discrete *(adjective)*. Separate. "One of the MCM problems involves discrete mathematics."
discreet *(adjective)*. Respectful of privacy. "You should be discreet when discussing private matters."

principle *(noun)*. A fundamental rule. "One of the principles of good writing is to be concise."
principal *(adjective)*. Primary. "Poor writing is the principal reason that MCM papers do not make it past the first round."

lose *(verb)*. Fail to keep. "You will lose your book if you do not write your name in it."
loose *(adjective)*. Not tight. "You can untie a knot if it is loose enough."

affect *(verb)*. Influence. "If you change the value of a parameter, it will affect your model."
effect *(noun)*. Result of an action. "The effect of not studying for a test is a failing grade."

ensure *(verb)*. Make certain. "You can use a spellchecker to ensure there are no misspelled words."
insure *(verb)*. Put an insurance policy on. "You should insure any valuable jewelry."

its *(noun)*. Possessive form of "it". "A dog will chew its toys."
it's *(contraction)*. "It is". "If a dog is hungry then it's going to eat."

However, it is probably best not to use contractions in formal writings, particularly when contractions could cause confusion.

The correct way to write out "can't" is "cannot", not "can not". Thus, you should say

Good: He cannot complete the task in time.

or

Good: He can't complete the task in time.

but not

Bad: He can not complete the task in time.

4 Use unobtrusive pronouns

English does not have a third person singular indefinite pronoun. Historically, the male pronoun has been used, such as "he" in the sentence "If an observer touches the experiment, then he will affect the outcome of the experiment." There are people who object to using the male pronoun as the default and there have been several attempts to introduce a new set of pronouns for third person indefinite. None are commonly used.

One unobtrusive solution to this problem that has historical justification is to use "they" for singular as well as plural. You could write "If an observer touches the experiment, then they will affect the outcome." and "A student should be careful not to lose their books."

5 Use articles ("the", "a" and "an") properly

The proper use of articles is not a simple matter, particularly to the Chinese people, for the Chinese language does not require an article before a noun. As a result, Chinese students tend to omit articles when they should be there, or use "the" for almost any noun. If there is any doubt about whether you are using an article correctly, you should consult a book on English usage or look for a similar construction in a known piece of good writing.

If you are using an article before a noun, you should use the definite article "the" if the noun refers to a unique object, otherwise you should use the indefinite article "a" or "an". For example, if you are talking about a differentiable function, you can refer to "the derivative", since there is only one derivative, and "an antiderivative", since there is more than one antiderivative.

In the case of non-unique objects, you should use "a" if the word *sounds* like it begins with a consonant, otherwise you should use "an". For example, you could refer to "a norm", since "norm" begins with an "N" sound, but you would refer to "an ℓ_1 norm", since "ℓ" begins with an "E" sound. For nouns beginning with the "u" sound, you should think of it as "you" that begins with "y", and so "a" instead of "an" should be used. Thus, we write "a European company" instead of "an European company", and we write "a US team" instead of "an US team".

Whether or not you should use an article before a noun is a bit trickier, but there are a couple things to keep in mind.

You should use an article before a noun which represents a single object.

You would never say "Function is differentiable", you would say "The function is differentiable" or "A function is differentiable".

You should not use an article before a noun which is supposed to represent all objects of its type. If you want to note that (in general) the derivative of a function is not necessarily continuous, you might say "Derivatives are not necessarily continuous." However, if you want to discuss certain derivatives, such as the derivatives of a certain group of functions, you could say "The derivatives are not necessarily continuous."

6 Useful verbs

We provide a list of verbs that could come in handy when writing a paper on mathematical modeling. For example,

> *You may need to investigate the problem from different points of view, survey what has been done in the literature, explore different ideas, formulate and justify your assumptions, design a model, device an algorithm, carry out numerical simulations, and compare one approach with another one.*

Compare to and compare with When you intend to juxtapose two or more items to illustrate differences, similarities, or both, use "compare with". If the differences are important, you should definitely use "compared with". For example,

> *We compare our results with the existing ones. We show that our method is more robust and our algorithm is faster.*

When you intend to assert similarities, use "compare to". For example,

> *We can compare her model for loading a railroad car to his method for loading a ship cargo.*

Study and investigate The word "study" can be used both as a verb and as a noun. Thus, we can say

> *A 24-hour study to investigate this problem suggests a new direction.*

Similarly, we can also say

A 24-hour investigation to study this problem suggests a new direction.

Seek and explore These two verbs are useful when describing your thoughts. For example,

We seek to devise a new model for solving the problem by exploring the new direction suggested by their investigations.

Design and devise These two verbs are useful when describing your solution. For example,

Based on our analysis, we design a model for the problem using integral linear programming. We then devise a polynomial-time approximation algorithm to produce near-optimal results.

Tackle and solve These two verbs are useful when describing your accomplishment. For example,

We tackle the problem using the new technique we developed in the previous section. While it is difficult to solve the problem completely, we are able to solve a major subproblem.

Approach and propose These two verbs are useful when describing a proposal. For example,

We approach the problem using the proposed method.

Note that "approach" can also be a noun.

We propose a new approach to tackling the problem.

We can also say

We propose a new approach to tackle the problem.

Chapter 5 Revising

After you finish writing your paper, you will need to reread it, paying close attention to content and style. You should be prepared to make minor adjustments and also rewrite parts of it. You should probably have a style guide and an English usage guide handy while you do this.

While you're reading, you should

- look for passive sentences; make them active if at all possible.
- look for complicated words that can be replaced by simpler words.
- look for complicated sentences than can be replaced by simpler sentences, possibly by breaking up the original sentence into two or three smaller sentences.
- make sure that all of your assumptions are explicitly justified.
- make sure that you present all of your information in logical order.
- make sure that any parts of your paper that you took from other sources are properly credited.

You should begin with your summary, which is where the judges will begin. Read each word and each sentence carefully. Remove information that is irrelevant, and make sure that everything that is relevant is included in the summary in a logical order. Be prepared to restructure your summary if it helps.

Next, you should skim your paper, and make sure that the parts of your paper that you highlighted (the section titles, the displayed equations, the tables and figures, and sentences put in bold font) give the reader a good idea of what is happening in your paper. Make sure that irrelevant information is not highlighted.

Finally, go through the entire paper. Although you will not be able to reexamine your paper as carefully as you did the summary, you should look for the same things.

This chapter provides examples of changes to actual papers to show what sort of changes could be made.

1 Example: Revising a title

One of the problems in the 2010 contest was to explain the sweet spot on a baseball bat. One of the papers submitted for this problem was "Science in Sweet Spot" [12]. We use this paper to explain how to revise titles and summeriness.

The title of your paper should be carefully chosen. The title of this paper consists of the following three errors:

1. It uses an inappropriate preposition. Prepositions are tricky, and the preposition here should probably be "of", not "in".
2. Since "spot" refers to a Hnique point, it should get an article, and in this case there should be a "the" before "spot".
3. Science in general is not being studied in this paper, but the particular science that is associated with the sweet spot is. So it should be "the science".

The title should probably be "The Science of the Sweet Spot".

2 Example: Revising a summary

Just as the summary should take up a disproportionate amount of your writing time, it should also take up a disproportionate amount of your revising time. As well as stylistic considerations, effort should be made to shorten it without losing any impact.

As an example of revising a summary, consider the summary of the same paper "Science in Sweet Spot" [12], which begins:

> *This paper mainly studies the 'sweet spot' on a baseball bat. Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball. Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model. With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively. By the law of kinetic energy conservation, hitting a baseball at different spots of the bat can be seen as an imperfect elastic collision of two balls. Thus given a bat, for different speeds of the coming ball and that of swinging the bat, the speed of the hitting ball can be calculated according to the Newton's collision law.*

Revising

1. *This paper mainly studies the "sweet spot" on a baseball bat.*
   A summary should not start off describing what "the paper" does; it leaves the authors and readers as outside observers. The paper does not do anything but describe what the authors did, the first sentence should begin with "We". While the authors may do other things besides study the sweet spot, the word "mainly" is superfluous and should be removed. Finally, "sweet spot" is a standard term for a particular part of the bat; unless the authors are suggesting that it is an inappropriate term, it should not be in quotes. The first sentence could be rewritten

   We study the sweet spot on a baseball bat.

   which is not only shorter, but also more dynamic. Compare this to the first sentence of "The Sweet Spot: A Wave Model of Baseball Bats", one of the Outstanding Winners:

   We determine the sweet spot on a baseball bat.

2. *Firstly, by analyzing the video of a professional hitter, a double-pivot swing (the arm and the finesse) model is established to describe a batter hitting a baseball.*
   This sentence is passive and should be made active. As written, the video is analyzed and the model is established, but the author and reader are left out. Notice that there are probably many videos of professional hitters, since no particular one has been established, the summary should refer (at least the first time) to "a video" rather than "the video". Finally, the word "firstly" is considered obsolete by many Americans. While this attitude is not universal, you would probably be better off by replacing "firstly" by "first" whenever it appears.
   This sentence could be replaced by

   We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter.

3. *Then based on the law of energy conservation, the double-pivot swing model is further transferred into a single-pivot swing model.*
   This sentence continues the passive tone of the summary. All of the passive

sentences should be rewritten to be active.

4. *With the given shape of the bat and the parameters for the single-pivot swing model, the moment of inertia of a wooden bat, corked bat and aluminum bat is calculated, respectively.*
   This is the first mention of the shape of the bat, nothing about it has so far been given and it is not mentioned again in the summary. Similarly, "the" parameters have not been mentioned before and are not mentioned again in the summary. These should not be mentioned here. Since they are used to create the model, they can be replaced by "Using our model ...". There is also no need yet to divide the types of bats into wooden, corked and aluminum.

Recall, you should not just revise your paper line by line; you also need to be willing to change your paper on a larger scale. The point of the summary is that hitting a ball with a bat is equivalent to hitting a ball with another ball, which the authors later refer to as the "hitting ball", and so the summary could be replaced by

> *We study the sweet spot on a baseball bat. We begin by establishing a model of a bat hitting a ball by analyzing a video of a professional hitter. Using this model, we determine that hitting a ball with a bat can be seen as an imperfect collision of two balls, where the bat is replaced by a "hitting ball".*

3 Example: Revising an introduction

One of the problems in the 2008 contest involved developing "an algorithm to construct Sudoku puzzles of varying difficulty." One of the papers submitted for this problem was "The Solvers' Sudoku or computers' Sudoku" [13]. We first note that there are two errors in the title. First, the "C" in computer should be capitalized. Second, there should be a "the" before "Computer". Thus, the title should be "The Solver's Sudoku or the Computer's Sudoku".

The paper is described in its introduction.

> *Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan. It is a simple game of logic and so easy to learn. It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it. Partial people began to study solving the puzzles by computer. However, generating Sudoku puz-*

zles is even more difficult when difficulty levels and a unique solution both are considered. It is also the task we are required to finish.

The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed. Difficulty levels are greatly depended on the metrics developed. The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution. Develop a series of metrics to define the the difficulty levels. Judge its difficulty level utilizing metrics.

Metrics are very important in the whole course of constructing puzzles. In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively. The former is subjective and the latter is impersonal.

Revising

1. *Sudoku (Japanese: sūdoku) is derived from Switzerland, developed in America and carried forward in Japan.*
   This is a catchy first sentence. However, the parts of the sentence are not parallel; "is derived", "developed" and "carried forward" do not all have the same structure. Also, saying that sudoku was "is derived from Switzerland" implies that the country itself was transformed into the game, the phrase "originated in" would be more accurate.
   If sudoku puzzles were relatively unknown, then the introduction should start with a sentence briefly stating what they are. The authors of this paper rightfully assume that sudoku puzzles are familiar to the readers.
2. *It is a simple game of logic and so easy to learn.*
   This sentence is simple and straightforward, but "so easy" is an idiom meaning "very easy". The authors probably want "so" to be used to indicate a conclusion; "hence easy to learn" or "so it is easy to learn" would be clearer.
3. *It is also a fun and addictive game and is puzzling millions of players all over the world. Many people like it.*
   Beginning too many sentences with "It is . . . " can be dull, and since the game is fun and played by millions, it is safe to assume that many people like it. The last sentence can be removed.
4. *Partial people began to study solving the puzzles by computer.*
   This is an unfortunate sentence; the thought of partial people doing anything is unsettling. It is probably a misedit from a draft of the paper. The authors

of the paper probably wanted to say that some people began to use computers to solve Sudoku puzzles.

5. *However, generating Sudoku puzzles is even more difficult when difficulty levels and a unique solution both are considered.*
   This does not flow from the previous sentence. "However" indicates that the difficulty of generating the puzzles is being contrasted with something, but it is not clear what that something is. There is also no indication what it is "even more difficult" than. Also, two similar words, such as "difficult" and "difficulty" should not be used close together.
6. *It is also the task we are required to finish.*
   This is passive and should be made active.
7. *The aim is to create a reliable sudoku algorithm which has least complexity and a unique solution should be guaranteed.*
   The authors should bring themselves into this sentence; it should begin "Our aim ...". The phrase "sudoku algorithm" is ambiguous; the reader can infer that the authors mean an algorithm to create the puzzles, but it would not hurt to mention that explicitly. The phrase "has least complexity" can be simplified to "is simplest". Finally, "a unique solution should be guaranteed" is passive and should be active.
8. *Difficulty levels are greatly depended on the metrics developed.*
   The phrase "depended on" should be "dependent on". This sentence is also the first mention of metrics, but it gives the impression that "the metrics" are already familiar to the reader. Either the authors should bring up metrics before this sentence, or make it clear they are introducing metrics here.
9. *The basic thought is that we delete digits from a solved puzzle, and then check out if there is a unique solution.*
   The authors did not indicate who is doing the thinking; the sentence would be better off starting with "Our basic thought". However, since they want to discuss what they are doing, "Our approach ..." would be a better start.
10. *Develop a series of metrics to define the difficulty level. Judge its difficulty level utilizing metrics.*
    These are not sentences. The authors probably intended the previous sentence to begin a list of items in their approach, but in that case the items should be in a bulleted list and they should be parallel. Here the items being listed are not parallel, and so they should be described separately with complete

sentences.

11. *Metrics are very important in the whole course of constructing puzzles.*
If something is important and it is not obvious to the reader, the authors should say why it is important. A statement of the form "X is important" is typically unnecessary and should be removed.

12. *In our paper, we develop metrics fit for persons' thought and metrics aimed at computers' "thought" respectively.*
The introduction is about the paper; the phrase "In our paper" is unnecessary and should be removed. Since two lists are not being compared, the sentence should not end with "respectively". Finally, the metrics should work for "a person's thoughts" and "a computer's 'thoughts'".
Based on the discussions given above, this introduction could be rewritten as

Sudoku, originated in Switzerland, was developed in America and popularized in Japan. It is easy to learn, and millions of players enjoy being puzzled by it. While some players have created algorithms to solve sudoku puzzles, our task is to create an algorithm to generate sudoku puzzles of various difficulty levels with unique solutions.

Since our task involves measuring the difficulty of the puzzles our algorithm generates, we develop a series of metrics to define and measure the difficulty. We create metrics to measure the difficulty of a puzzle for a person and metrics to measure the difficulty of a puzzle for a computer. Our approach is to

- *start with an already solved puzzle;*
- *delete some digits;*
- *check to see if there is a unique solution;*
- *measure the difficulty level.*

4 Example: Revising assumptions

One of the problems in the 2011 contest involved comparing conventional and electric cars. One of the papers submitted for this problem was "Can Electric Vehicle Be Widely Used" [14]. Since the paper is about more than one electric vehicle, the title should probably be "Can Electric Vehicles Be Widely Used". If by "electric vehicle" the authors mean the entire class of such vehicles, then the title should be "Can the Electric Vehicle Be Widely Used."

The paper discusses three types of vehicles: conventional vehicles (CV), electric vehicles (EV) and hybrid-electric vehicles (HEV). The second section of the paper is a list of the assumptions the authors will use, given nicely as an enumerated list.

1. *We will select one vehicle mode to represent CV, EV and HEV.*
2. *We assume there is no difference in performance of each vehicle type.*
3. *We select France, USA, and China to represent European countries, American countries and Asian countries.*
4. *We assume performance of CV, EV and HEV will not change in the future.*
5. *Based on common sense, we assume each vehicle travels 10,000 km every year.*
6. *We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.*
7. *We do not consider the dissipation during energy conversion.*

The assumptions here, whether reasonable or not, are not given explicit justifications.

Revising

1. *We will select one vehicle mode to represent CV, EV and HEV.*
   The authors mean "vehicle model", not "vehicle mode". The way this assumption is phrased, it sounds as if the authors are choosing a single model to represent three different types of vehicles; they mean to say that each type of vehicle will be represented by a different model. Finally, choosing one vehicle to represent a class of cars is not an assumption, but part of the authors' approach to the problem.
2. *We assume there is no difference in performance of each vehicle type.*
   "Performance" could mean a lot of things; what type of performance is being considered should be mentioned explicitly. Also, while the authors mean that all conventional vehicles perform similarly, all electric vehicles perform similarly, and all hybrid-electric vehicles perform similarly, the statement they make could be interpreted as meaning that conventional vehicles perform similarly to electric vehicles, and both of those perform similarly to hybrid-electric vehicles. Neither interpretation of this assumption is reasonable, however, and either one would require an explanation of why the authors are making the assumption. Here, the assumptions are purely for simplifying the problem and that should be stated explicitly; if the authors have evidence that their assumption is reasonable they should provide it.

3. *We select France, USA, and China to represent European countries, American countries and Asian countries.*
   The authors do not mean that all of the countries listed represent all of the continents listed, they mean that each country listed represents the corresponding continent. The second list should end with "respectively" to indicate this. Still, this item is not an assumption but rather the way the authors are approaching the problem. Their assumption would be that these choices are reasonable. If there is evidence that the countries are typical of their continents, that should be stated with the assumption.
4. *We assume performance of CV, EV and HEV will not change in the future.*
   Given the pace of technology, this is probably wrong. It may be a necessary simplifying assumption, but that reason should be stated explicitly.
5. *Based on common sense, we assume each vehicle travels 10,000 km every year.*
   You cannot conclude how far each type of car will travel per year from common sense; the authors should either state this as a simplifying assumption or they should provide data to support this distance.
6. *We will not specify different types of power plants and assume that each plant will generate 10,000,000 kwh annually on average.*
   They should provide data to support this assumption.
7. *We do not consider the dissipation during energy conversion.*
   This is not stated as an assumption; it should be "There is no dissipation during energy conversion." While not a reasonable assumption, it should be stated as a necessary simplifying assumption and should be mentioned when they discuss weaknesses of their model.

The section on assumptions for this paper could be rewritten to something like:

> *We will choose the 2010 Ford Focus 1.4 Duratec to represent conventional vehicles, the Tesla Roadster to represent electric vehicles, and the Chevrolet Volt to represent hybrid-electric vehicles. We will also choose France as a representative European country, the USA as a representative American country, and China as a representative Asian country.*
>
> *We will make the following simplifying assumptions.*
>
> 1. *Each car we chose is typical of its class.*
> 2. *Vehicle performance will not change in the future.*
> 3. *Each vehicle will travel 10,000 km per year.*

4. *Each country we chose is typical of its continent.*
5. *Each power plant will generate an average of 10,000,000 kwh per year.*
6. *There is no energy dissipation during energy conversion.*

The authors should also clarify what they mean by "performance". If the authors could have gotten data to support any simplifying assumptions, that assumption should be mentioned in a different list with the supporting information. Finally, the simplifying assumptions they make should be mentioned as weaknesses when they discuss the strengths and weaknesses of their model.

Chapter 6 Symbols and Figures

Any paper involving a substantial amount of mathematics, such as your MCM paper, will need to use mathematical symbols. Moreover, papers on mathematical modeling may also use figures and tables. Symbols should be used sparingly, however; they should only be used when necessary. This chapter discusses some conventional rules and suggestions for using mathematical symbols, figures and tables.

1 Typography

Good typography can be important in making sure that the paper is easy to read. The font size should be a comfortable size, you should use 11 or 12 point base font. Since font changes can be distracting, you should probably stick with one serifed font for text (as well as a font for math). The style and size of the font can change in limited ways; bold for emphasis, large for section headings, italic for book titles, etc., but otherwise the font should be fixed. For example, it is conventional to use the Times New Roman for text, and its italic form for math. The section and subsection headings should stand out distinctly from the text of the paper; a brief flipping through the pages should allow easy perusing of the section titles.

You should make sure that whatever software you use to typeset your paper has enough math symbols. For example, you should not have to use the Greek letter ϵ for set inclusion; you should be able to write $x \in X$ and not $x \epsilon X$.

2 Avoid using symbols in titles

Since titles are short, there is no way to define symbols in a title. Thus, using symbols in a title may puzzle the reader, unless the meanings of the symbols are well understood by the general audience.

3 Stick with notational and typographical conventions

Mathematics involves a lot of conventions which are not formal rules but still make it easier for the reader to follow what you are doing. For example, mathematical symbols should be typeset in italics. Thus, instead of

Bad: It requires that I be a constant for ... to be true.

you should say

Good: It requires that I be a constant for ... to be true.

Otherwise, the sentence could cause confusion that the author who wrote the sentence is required to be a constant. "I" here is a mathematical symbol; although it may not be the best choice for a symbol, writing it in italics will eliminate any possible confusion. For another example, matrices are usually represented by capitals in math bold (or italic capital in bold) and the entries of a matrix are usually represented by the corresponding lower case letter in italics. If you referred to the matrix a and the entry A_{ij}, for example, it would give the reader reason to pause; you should be using the matrix A and the entry a_{ij}. Similarly, the Greek letter ϵ typically represents a small positive number. Using ϵ to represent a large number would make the reader look twice and interfere with the flow of the reading.

Be sure to keep in mind that mathematical symbols are case sensitive, and even emphasis sensitive. The symbols X, $\mathbf{X}$, x and $\mathbf{x}$ are all different. In math mode, standard typographical conventions (such as upper case for sets, lower case for elements, bold for vectors and matrices) should be used to help the reader follow your work. While most functions and variable are typeset in math italics, such as $f(x)$, standard math functions are typeset in upright Roman, such as $\sin(x)$ and $\max\{a, b, c\}$.

4 Do not reuse symbols

Each symbol in your paper should be used for only one purpose. Do not use Δ to represent a matrix in one part of your paper and then use it for the discriminant of a polynomial in another part. Similarly, if you use a sigma notation for a sum, then you should not use Σ as a variable anywhere else in your paper. If you are using complex numbers in your paper, you should avoid using i as an index variable.

5 Do not use symbols for words

Since you are not writing a paper on symbolic logic, you should not write like you are. Instead of writing

Bad: $x = 2 \Rightarrow x^2 - 4 = 0$

you should write something like

Good: Since $x = 2$, *we know* $x^2 - 4 = 0$.

Likewise, you should write out "for all" and "there exists" and not use the symbols $\forall$ and $\exists$.

Similarly, you should not use symbols to take the place of words in an otherwise symbol-free sentence. You should never write something like

Bad: The two functions are $=$.

it should always be

Good: The two functions are equal.

6 Spell out numbers that are used as adjectives

Switching numerals to words can slow down reading, so if a number represents an adjective it should be written out. Instead of

Bad: There are 3 solutions.

you should write

Good: There are three solutions.

This does not apply to numbers being used as parts of units; in the sentences

Good: The rod is 5 feet long.

Good: It is a 5-foot-long rod.

the "5" should be left as a numeral.

7 Avoid unnecessary subscripts

Whenever possible, use x and y instead of x_1 and x_2. For example, it is often tempting to let $x_1, x_2 \in X$ when mentioning two elements of a set X, but unless the actual subscripts are used, you should let $x, y \in X$.

You should also avoid inconsistency in your use of subscripts. For example, if you are using x_1 and x_2, then a linear combination should be written $a_1x_1 + a_2x_2$ rather than $ax_1 + bx_2$. Likewise, if you are using x and y, then a linear combination should be written $ax + by$ rather than $a_1x + a_2y$.

8 Be consistent with your notation and terminology

Even though the expressions x_i for $i = 1, 2, 3, \dots$ and x_j for $j = 1, 2, 3, \dots$ are the same sequence, using different letters for the subscripts make them look different. You should consistently use the same letter for the same subscripts.

The use of inconsistent terminology often go unnoticed but can be puzzling to the reader. For example, do not say "Cholesky factorization" in one part and then "Cholesky decomposition" in another part; do not refer to $ker(A)$ as the null-space or $null(A)$ as the kernel.

9 Keep subscripts in the same order

When repeating pairs of subscripts, you should try to keep them in the same order. For example, when describing a decreasing sequence you should avoid

Bad: $x_i < x_j$ *when* $j < i$.

instead you should say

Good: $x_i < x_j$ *when* $i > j$.

The first sentence, while mathematically correct, looks awkward, which may cause the reader to pause.

10 Remove symbols that are used only once

If a symbol appears only once, there is no reason for it to appear. If you write

Bad: A differentiable function f is continuous.

the f is typically superfluous; you should write

Good: A differentiable function is continuous.

11 Figures and tables

Displaying figures and tables is an efficient way to quickly and clearly express data. Most people grasp information visually easier than textually. Many MCM papers use graphs to compare the results of their models; the reader can easily see the effect of changing the parameters.

For a small data set, tables are a better way of showing the information. For larger data sets, some type of graph would be more appropriate, but care should be taken to make the graphs as simple as possible while still getting across the desired information. A simple figure will help the reader through your paper, a complicated figure can slow the reader down.

Many MCM papers include figures but fail to discuss them in the paper itself. The judges view this unfavorably. While the reader may be able to determine the relevance of the figure, you should not make the reader work at it. The figure, as well as the table, should have a label, such as "Figure 2.1" or "Table 2.3", so that you can refer to it when you discuss it in the main text of your paper.

Each figure should have a caption; the caption should describe the figure so that even someone who has not read the paper, or someone who is simply skimming the paper, will understand what the figure represents. The figure itself should also have relevant labels; graphs should have the axes labeled with what information they represent and what information is referred to in the paper, legends should be used when appropriate, and tables should have the rows and columns labeled.

Chapter 7 Math and Sentences

In any paper involving mathematics, the math should flow smoothly through the narrative. Your paper should be written in sentences, including the mathematics. Thus, in addition to using symbols, figures, and tables appropriately, you must also make the math and the narrative coherent, and you must make sure that they are clearly distinguishable using typography. We will discuss how to write coherent sentences that contain math.

1 Do not begin a sentence with a symbol

A sentence beginning with a symbol looks awkward. Instead of writing

Bad: f is differentiable.

you should put a description before the symbol f,

Good: The function f is differentiable.

If the reader has not seen the symbol f in a while, a reminder could be helpful,

Good: The exponential function f is differentiable.

2 Make sure the same symbol is always read the same

If you use a symbol twice in the same sentence, you should be able to read it the same way both time. For example, the sentence

Bad: For $x = 2$ we know that $x^2 = 4$.

is read "For x equal to two, we know that x squared is equal to 4." The symbol $=$ has two different readings in the same sentence; first it is read as "equal to" and then it is read as "is equal to". This sentence should be

Good: If $x = 2$ *then* $x^2 = 4$.

which is read "If x is equal to 2 then x squared is equal to 4", and so the symbol $=$ is consistently read as "is equal to".

3 Punctuation and coherence

Any mathematical expressions that are part of the paper are part of a sentence and should be punctuated appropriately. The paper should sound good when read out loud; in fact, you should probably read it out loud after you have written it. This is an effective method to find out whether your writing is coherent.

A sentence like "If $x > 0$ then $f(x)$ is differentiable" would be read "If x is greater than zero, then f of x is differentiable." Note that an equation or inequality can be a noun in a sentence or the subject, verb, and object in a sentence. The sentence "The equation $x^2 = 1$ has two solutions" is read aloud as "The equation x squared equal to one has two solutions", where the equation $x^2 = 1$ is a noun. The sentence "If $x = 1$ then $x^2 = 1$" is read aloud as "If x is equal to one then x squared is equal to one", where the equation $x^2 = 1$ consists of a subject (x^2), verb ($=$), and object (1).

4 Separate different formulas with words

Two symbols from different expressions should be separated by words, in addition to punctuation. For example, instead of

Bad: Let x_i, $i = 1, 2, 3, \ldots$ *be a sequence.*

you should write

Good: Let x_i *for* $i = 1, 2, 3, \ldots$ *be a sequence.*

or

Good: Let x_i *be a sequence, where* $i = 1, 2, 3, \ldots$.

Instead of

Bad: Since $a = 2$, $a^2 = 4$.

you should write

Good: Since $a = 2$, we have $a^2 = 4$.

5 Use simple English

Even though your paper will involve a lot of mathematics, it should primarily be written in simple English. Nobody, including mathematicians, like to read jargon-heavy papers. Some technical terms will be necessary, but they should be kept to a minimum.

6 Do not overload math terms

Rule 12 in Chapter 3 says that we should void using the same word for different purposes. This rule also applies to mathematical terms. For example, the term "element" may often be overloaded in mathematical writing. If $\mathbf{x} = (x, y, z)$ is a vector in $\mathbb{R}^3$, then x is sometimes referred to as an element of $\mathbf{x}$. But then x is called an element of $\mathbf{x}$ and $\mathbf{x}$ is an element of $\mathbb{R}^3$. The word "element" means two different things here; it would be better to use two different words. In this case, x could be called a *component* of $\mathbf{x}$, for example.

7 Every "if" should have a "then"

Too often, the "then" in an "if ... then" statement is left unstated, the conclusion of the statement is implied. The conclusion of such a statement is sometimes easy to imply and sometimes hard. For example, the conclusion of the following statement is easy to imply.

Bad: If it rains, the grass will get wet.

It is better to make sure that your conclusions are explicit, particularly in math papers.

Good: If it rains, then the grass will get wet.

But the following statement

Bad: If $x = 1$, $y = 2$, $x + y = 3$

is poor not only because the equations are not separated by words, but the hypotheses are not clearly separated from the conclusion. The statement should be

> *Good: If* $x = 1$ *and* $y = 2$*, then* $x + y = 3$.

Otherwise, the sentence may also be interpreted as "If $x = 1$, then $y = 2$ and $x + y = 3$."

You should always make sure that your conclusions are explicit. Simply including a "then" for every "if" will do it.

8 Give the reader useful reminders

Try not to make the reader tax their memory or reread previous parts of your paper. You will base a lot of your work on the assumptions you make at the beginning of your paper; whenever you use one of your assumptions be sure to tell the reader which one you are using. For example, instead of writing

> *Bad: By our assumptions, we can conclude that* f *is continuous.*

you should write

> *Good: Since we assumed that the density function is continuous, we can conclude that* f *is continuous.*

Recalling that symbols should be given reminders themselves, you should actually write

> *Good: Since we assumed that the density function is continuous, we can conclude that the function* f *is continuous.*

or, better yet,

> *Good: Since we assumed that the density function is continuous, we can conclude that the mass function* f *is also continuous.*

When you refer to a variable that has already been defined, you should remind the reader what sort of object it represents. Instead of writing

> *Bad: Hence* A *is invertible.*

you should write

Good: Hence the matrix A is invertible.

9 Define terms right before you use them

The reader of your paper should not have to deal with several new terms at one time. There should not be a paragraph with several definitions in it, instead new terms should be defined right before they are used. For example, instead of writing

> *Bad: A function is **smooth** if it infinitely differentiable. A function is* $\mathbf{C}^1$ *if it has a continuous derivative. A function is ...*

You should write

> *Good: A function is **smooth** if it is infinitely differentiable. Suppose that f is a smooth function.*

New terms should also be in bold font when they are first defined.

You should only define a term if it is used more than once; if the only reason you define "smooth" is to mention that the function f is smooth, you should skip the definition and write

> *Good: Suppose that the function f is infinitely differentiable.*

10 Use displayed equations appropriately

Recall that a mathematical expression in a paper can be inline, where it simply appears on a line with the rest of the text, or displayed, where it appears on its own line, usually separated from the surrounding text by some vertical space. The equation $x^2 - 1 = 0$ is inline and the equation

$$x^2 - 1 = 0$$

is displayed. In both cases the expression is part of a sentence and needs to be treated as such, including appropriate punctuation.

A displayed equation will stand out from the rest of the text, and so should be used when you want to emphasize it. A displayed equation should only be numbered when it will be referred to later in the paper.

Mathematical expressions, inline or displayed, are parts of sentences and should be punctuated appropriately. The text before the expression normally does not get any special punctuation, and you should not put a colon there. Thus, instead of writing

> *Bad: Thus, a simpler form of double Pareto distribution density function is:*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \le 1 \\ x^{-\alpha-1}, & x \ge 1 \end{cases}$$
>
> *for some $\alpha > 0$ and $\beta > 0$.*

You should write

> *Good: Thus, a simpler form of double Pareto distribution density function is*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \le 1 \\ x^{-\alpha-1}, & x \ge 1 \end{cases}$$
>
> *for some $\alpha > 0$ and $\beta > 0$.*

But if you use the word "following", "below", or the phrase "as follows", then do use a colon to separate the text from the expression. For example, write

> *Good: Thus, a simpler form of double Pareto distribution density function is as follows:*
>
> $$f(x) = \frac{\alpha\beta}{\alpha+\beta}\begin{cases} x^{\beta-1}, & 0 < x \le 1 \\ x^{-\alpha-1}, & x \ge 1 \end{cases}$$
>
> *for some $\alpha > 0$ and $\beta > 0$.*

11 Make your fractions readable

Fractions can be written with a slash (x/y) or a horizontal bar ($\frac{x}{y}$). For inline mathematics, and other places where vertical space is limited (such as in subscripts and superscripts), writing fractions with a slash is probably better. If a fraction is somewhat complicated, however, using a slash might make the fraction hard to read, in which case a horizontal line would be better.

12 Watch your breaks and alignment in expressions

An inline expression should not be broken across lines. If an expression is too long, then display it. If a displayed expression is broken across lines, the line break should occur at a low precedence binary operation; break the expression at a plus or minus if possible. Often the binary operation appears on the second line when this is done, such as

$$\begin{gathered} abcdef + ghijkl + mnopqr + stuvwx \\ +yz. \end{gathered}$$

There should not be a page break in the middle of a displayed formula. If a math expression is too long and needs to go over to the next page, then you should use appropriate words to separate the expression at the end of the current page, and continue to write the rest of the expression in the next page.

If a displayed expression consists of a series of equalities, then the equalities should line up at the equal signs; the result should look like

$$\begin{aligned} a + b &= c + d, \\ e &= f + g + h, \\ i + j &= k. \end{aligned}$$

Similarly, a string of equalities should line up at the equal signs, such as

$$\begin{aligned} a + b &= c + d \\ &= e + f + g \\ &= h. \end{aligned}$$

13 Align your ellipsis properly

If the middle part of a sequence is left out, it is replaced by a symbol consisting of three dots, called an ellipsis. The ellipsis should be aligned along the baseline, as in

$$x_1, x_2, \ldots, x_n.$$

There should be commas on either side of the ellipsis. If the middle part of a sum is left off, as in

$$x_1 + x_2 + \cdots + x_n,$$

then the ellipsis should be centered, and there should be a plus sign on both sides. The same rule apples to other operations.

Chapter 8 LaTeX

LaTeX is a commonly used math formatting program. Your paper needs to be submitted both in paper form and electronic form. The electronic version is required to be in either Adobe PDF format or MS Word. LaTeX can produce nice-looking output and many MCM coaches recommend using LaTeX to produce PDF.

LaTeX is not a WYSIWYG (What You See Is What You Get) word processor, like MS Word; it's more of a markup language, like HTML. LaTeX is very configurable, but you should not worry about that while you are working on your MCM paper. You should just use the basics of LaTeX on your MCM paper and save any fancier typography for some other occasion. This chapter provides you with a very short introduction to LaTeX. It covers just enough of the mechanics to allow you to write an MCM paper but omitting some commands and options you probably will not need. If you want to learn more about LaTeX, a good place to start is *The Not So Short Introduction to LaTeX 2_ε* [15].

1 Compiling

When typing a LaTeX document, you should use a text editor, not a word processor like MS Word. If you insist on using a word processor, be sure to save your work as a plain text file. There are editors with extra features which will help you work with LaTeX and there are programs which allow you to work with LaTeX in a WYSIWYG manner, but this chapter will discuss how to work with LaTeX without these extra features. The authors recommend CTeX to the Chinese users, which can be downloaded free at `http://www.ctex.org` and includes a text editor WinEdt.

Once you have a document typed in, you should have it as a file with an extension of `.tex`, such as `mydocument.tex`. Then you need to compile it. If you have LaTeX installed properly you should be able to compile your file into a PDF document with the command

```
> pdflatex mydocument.tex
```

or on the tool bar of WinEdt, select option **TeX**, then **PDF**, and then **PDFLaTeX**. Compiling a LaTeX file is often called **latexing** it. (You may also use the `latex` command to compile `mydocument.tex` and generate a DVI file, and then produce a PDF file from it.) If your document uses any of LaTeX's automatic referencing features, you will need to latex it twice to get the references correct.

LaTeX commands typically start with a backslash (\) and are often followed by arguments enclosed in braces ({}).

When compiling your LaTeX file, you will probably see messages on the screen, and if there are no syntactic errors you will end up with a file `mydocument.pdf`. To view the result, you will need to use a PDF viewer; make sure you know the name of the viewer on your computer. You can then view your formatted document with a command like

```
> pdfviewer mydocument.pdf
```

If there are typesetting errors, they will need to be fixed before you get a PDF file. Errors are usually misspelled command names or unbalanced braces. For example, if you are supposed to type

```
\command{arg}
```

and you accidentally type

```
\commnd{arg}
```

or

```
\command{arg
```

you will get an error.

Working on a LaTeX document involves a typing, latexing, and previewing cycle. Repeat this cycle a few times to produce a satisfactory PDF file.

2 A very basic document

A very simple "Hello World" document would look like

```
\documentclass[12pt]{article}
\begin{document}
Hello, World!
\end{document}
```

If you save this as a file `mydocument.tex` and latex it, you will get a PDF file with the line "Hello, World!" near the top and the page number "1" at the bottom.

The first line,

```
\documentclass[12pt]{article}
```

states what kind of document is being created. In the above example, an article is being written; there are other options, such as "book", but for the MCM you will want to write an article. The `[12pt]` between the `\documentclass` and `{article}` is optional; without it the PDF file will have 10 point font size, with this option the file will have 12 point font size. For an MCM paper, you should use 11 point or 12 point font.

The space between

```
\documentclass[12pt]{article}
```

and

```
\begin{document}
```

is called the **preamble**. In this example the preamble is empty, but later you will see some useful commands to put in the preamble.

The actual text of the paper goes between the lines

```
\begin{document}
```

and

```
\end{document}
```

Ordinary text gets typed here, augmented by commands and environments. A command is typically given by a backslash followed by some text, such as `\ldots`. This command will insert a three dot ellipsis which is used to indicated that part of a sequence is omitted.

Input
`a, b, c, \ldots , z.`
Output
a, b, c, ..., z.

Some commands require an argument, which will be given between two braces ({}) following the command, such as `\textbf{text}`. This command will cause **text** to be typeset in bold face font.

Input
`This is \textbf{bold text}.`
Output
This is **bold text**.

An environment is the text between

`\begin{environment}`

and

`\end{environment}`

An environment will affect the enclosed text in some way. For example, the `center` environment will center the text.

Input

```
The following text
\begin{center}
is centered.
\end{center}
```

Output

The following text

is centered.

When you type your document, LaTeX will do the formatting. The spacing is taken care of by LaTeX, if you type several spaces (and in LaTeX, spaces and tabs are treated the same) between words, LaTeX will still space the words as it deems fit.

Input

```
Hello World!


Hello          World!
```

```
Hello
   World!
```

Output

Hello World!
Hello World!
Hello World!

Note that starting a new line in the input file does not start a new paragraph in the output file. To start a new paragraph, you will need to insert a blank line. If you want to start a new line without a new paragraph, you can end a line with \\.

Input

```
Here is the first sentence of a paragraph. Here is the
second sentence of a paragraph.\\
Here is the third sentence of a paragraph.

Here is the first sentence of a new paragraph.
```

Output

Here is the first sentence of a paragraph. Here is the second sentence of a paragraph.
Here is the third sentence of a paragraph.

Here is the first sentence of a new paragraph.

Any line beginning with a percent sign (%) is a *comment* line and is ignored by LaTeX.

Input

```
Hello world.
% Goodbye World.
Hello again.
```

Output

Hello world. Hello again.

You can doublespace your paper, if needed, by putting

```
\usepackage[doublespacing]{setspace}
```

in the preamble of your document.

You can also set different line space. For example, if you want space of 1.5 lines, you may do so by putting

```
\renewcommand{\baselinestrech}{1.5}
```

in the preamble of your document

3 Characters

LATEX can read any of the 128 ASCII characters. The number of characters you can type into a LATEX document can be increased to include non-ASCII characters by using the `inputenc` package; you can use the package by inserting the line

```
\usepackage[utf8]{inputenc}
```

into the preamble of your document.

With the `inputenc` package, you can insert accented characters into your paper; LATEX also provides a way to do that with the usual keyboard characters. For example, you can insert an umlaut over a character by preceding it with `\"`; `\"a` will result in "ä". Table 8.1 gives some more of the accents that LATEX can provide.

Table 8.1 Accents.

Input	Output	Input	Output	Input	Output
\`a	à	\'a	á	\"a	ä
\~a	ã	\^a	â	\c{c}	ç

Most characters that you put into a LATEX document will be typeset, but the characters \, #, $, %, ^, &, _, {, } and ~ have special meanings. If you want to have these characters show up in your document you will have to use special commands. The backslash can be entered with the command `\textbackslash`, the rest can be entered by using a backslash prefix.

Input
`\textbackslash \# \$ \% \^{} \& \_ \{ \} \~{}`
Output
\# $ % ^ & _ { } ~

The empty braces ({}) are necessary in some cases to prevent unwanted effects after the character.

You have already seen that the command `\ldots` can be used to insert an ellipsis into your document. Commands for some other special characters you can insert are given in Table 8.2. If you want to use the euro character, you need to have

```
\usepackage{textcomp}
```

in the preamble of your document. Note that the quote character `"` does not distinguish between left and right; LaTeX uses `` for left quotation marks and '' for right quotation marks.

Input

```
``Hello'' looks better than "Hello".
```

Output

“Hello” looks better than ”Hello”.

You have seen that the command `\textbf{text}` will typeset **text** in bold face. Similarly, `\textit{text}` will typeset `text` in italics and `\texttt{text}` will typeset `text` in a monospace typewriter font. If you want to insert a large block of text in monospace typewriter font, such as when you are inserting some computer code, you can use the `verbatim` environment. This will also inhibit the meanings of any special characters, so they can be inserted without commands.

Table 8.2 Characters.

Input	Output	Input	Output	Input	Output
`\ldots`	...	`` `` ``	“	`''`	”
`\textbackslash`	\	`\#`	#	`\$`	$
`\%`	%	`\^{}`	^	`\&`	&
`\_{}`	_	`\{`	{	`\}`	}
`\texteuro`	€	`\~{}`	~	`$\sim$`	∼

Input

```
To use the Euro symbol, you will want to have
\begin{verbatim}
\usepackage{textcomp}
\end{verbatim} in the preamble of your document.
```

Output
To use the Euro symbol, you will want to have `\usepackage{textcomp}` in the preamble of your document.

4 Mathematics

LaTeX was created to typeset mathematics well, and your MCM paper will necessarily include a lot of mathematics. The **amsmath** and **amssymb** additions to LaTeX help with the mathematics, so you should add
`\usepackage{amsmath, amssymb}`
to the preamble of your document.

Simple mathematics

Any mathematics in your document must be entered in *math mode*. For inline mathematics, math mode takes place between dollar signs. For displayed mathematics, math mode takes place between `\[` and `\]`, or between $$ and $$.

Input

```
\[\sum_{k=1}^n k = \frac{n(n+1)}{2}\] and
$$ \sum_{k=1}^n k = \frac{n(n+1)}{2}$$
have the same displaying effect.
```

Output

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

and

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

have the same displaying effect.

More complex displaying methods will be mentioned later.

Letters in math mode are typeset in math italics, and so any mathematics, even a single variable, should be entered in math mode.

Input
`You should refer to the variable $x$,` `not the variable x.`
Output
You should refer to the variable x, not the variable x.

If you want to use a bold math symbol, you should use the `\mathbf{math}` command.

Input
`Consider the vector $\mathbf{x} = (x,y,z)$.`
Output
Consider the vector $\mathbf{x} = (x, y, z)$.

Simple formulas and equations can be entered using keyboard characters:

Input
`If $a+b=c$ and $a>0$, then $b<c$.`
Output
If $a + b = c$ and $a > 0$, then $b < c$.

While basic functions are also typeset in math italics, named functions, such as sin and log, are typeset in upright Roman font. These functions have commands which are used to enter them in math mode; the commands are typically a backslash followed by the function name.

Input
`The function $f(x)$ is in math italic,` `but $\sin(x)$ is not.`
Output
The function $f(x)$ is in math italic, but $\sin(x)$ is not.

There are many characters used in math that are not available on a typical keyboard. LaTeX provides commands to enter these characters. A partial list of such commands and binary operators is given in Table 8.3. The `\circ` symbol can be used as a superscript to indicate degrees, 90°.

To insert a Greek letter in math mode, you can use the command beginning with a backslash followed by the spelling of the Greek letter. If the spelling is capitalized, then the result will be the capital Greek letter.

Input
`From $\alpha$ to $\Omega$.`
Output
From α to Ω.

Some capital Greek letters, like capital alpha and capital beta, are identical to English letters. In this case, LaTeX does not have a command for this Greek letter, the English equivalent must be used.

Input
`Capital alpha is $A$; capital delta is $\Delta$.`
Output
Capital alpha is A; capital delta is Δ.

Table 8.3 Common math symbols.

Input	Output	Input	Output
`\le`	$\le$	`\ge`	$\ge$
`\subset`	$\subset$	`\supset`	$\supset$
`\subseteq`	$\subseteq$	`\supseteq`	$\supseteq$
`\in`	$\in$	`\notin`	$\notin$
`=`	$=$	`\neq`	$\neq$
`\pm`	$\pm$	`\div`	$\div$
`\cdot`	$\cdot$	`\times`	$\times$
`\to`	$\to$	`\mapsto`	$\mapsto$
`\partial`	∂	`\infty`	∞
`\circ`	$\circ$	`\vdots`	$\vdots$
`\ldots`	$\ldots$	`\cdots`	$\cdots$
`\ddots`	$\ddots$		

Mathematical symbols (including Greek and Hebrew letters) supported by the **amssymb** package can be found at the following web page: **http://amath.colorado.edu/documentation/LaTeX/Symbols.pdf**.

Delimiters

You can group expressions together in math mode using parentheses. When you nest groupings, you may want to use different delimiters; you can also use brackets (`[]`) and braces ({}). Since braces are special characters, you will need to enter them with `\{` and `\}`.

Input
`$\{(a+b)\cdot c\}\cdot d$`
Output
$\{(a+b)\cdot c\}\cdot d$

When you are putting delimiters around a tall expression, you may wish to use larger delimiters to make it look better. LaTeX can automatically create appropriately sized delimiters. To enter left and right parentheses, you precede then with `\left` and `\right`.

Input
`The expression $(1+\frac{2}{3})$ may look okay,` `but $\left(1+\frac{2}{3}\right)$ looks better.`
Output
The expression $(1+\frac{2}{3})$ may look okay, but $\left(1+\frac{2}{3}\right)$ looks better.

Basic math constructions

Superscripts To create exponents and other superscripts, you can use the caret (`^`). In math mode, the caret will turn the next object into an exponent.

Input
`The expression $a$ squared is written as $a^2$.`

Output
The expression a squared is written as a^2.

If the exponent should include more than the next character, you will need to group the characters you want within braces.

Input
`Note that $a^23$ is not the same as $a^{23}$.`
Output
Note that a^23 is not the same as a^{23}.

Subscripts Subscripts are like superscripts, except they use the underscore (_) instead of the caret.

Input
`Again, $a_23$ is not the same as $a_{23}.$`
Output
Again, a_23 is not the same as a_{23}.

Superscripts and subscripts can be nested, but you should avoid doing that if at all possible.

Input
`The expression $a_{3^2}$ looks awkward,` `but is not as bad as $a_{i^{j_{k^l}}}$.`
Output
The expression a_{3^2} looks awkward, but is not as bad as $a_{i^{j_{k^l}}}$.

Certain operators have parts of their expression raised and lowered; for example, summation sums usually have their index and beginning index value lowered and the end index value raised. Sigma sums are typed in with the command `\sum`; other such operators are products (`\prod`), integrals (`\int`), and limits (`\lim`). The parts being lowered are entered as subscripts and the parts being raised are entered as superscripts.

Input
`$\lim_{n \to \infty}` `\sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) =` `\int_a^b f(x){\rm d}x$`
Output
$\lim_{n \to \infty} \sum_{k=1}^{n} ((b-a)/n) f(a + k(b-a)/n) = \int_a^b f(x){\rm d}x$

Fractions Fractions can be simply typed in with a slash.

Input
`Two thirds can be typed as $2/3$.`
Output
Two thirds can be typed as $2/3$.

If you want a fraction with a horizontal line, you can use the `\frac{top}{bottom}` command.

Input
`Two thirds can also be written $\frac{2}{3}$.`
Output
Two thirds can also be written $\frac{2}{3}$.

Binomial coefficients Binomial coefficients are similar to fractions, but use the command `\binom{top}{bottom}`.

Input
`The value of $\binom{6}{4}$ is $15$.`

Output

The value of $\binom{6}{4}$ is 15.

Displayed Equations

There are three reasons you might want to display an equation. The extra room given to displayed equations allows LaTeX to typeset a nicer looking equation. The extra room makes a displayed equation more noticeable, and so provides emphasis. Finally, displayed equations can be given labels that allow you to refer to them in other parts of the document.

An equation in an `equation` environment will be displayed and labeled. If you are not going to refer to it elsewhere you should not label it, for this you can use the `equation*` environment. The same commands that work for inline mathematics also work for displayed equations; the extra vertical space in displayed equations can be used to improve the appearance of the expression.

Input

```
The equations
$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,
\begin{equation}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8},
\end{equation}
and
\begin{equation*}
\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}
\end{equation*}
are all the same expression.
```

Output

The equations $\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$,

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}, \tag{8.1}$$

and

$$\sum_{k=1}^{3} \frac{1}{2} = \frac{7}{8}$$

are all the same expression.

If you want to include ordinary text in a displayed equation, you can use the `\text{text}` command.

Input

```
\begin{equation*}
x^2 \ge 0 \text{ for all real } x.
\end{equation*}
```

Output

$$x^2 \ge 0 \text{ for all real } x.$$

If you want to refer to a displayed equation at a different part of the document, you need to give it a label. To label an equation, put `\label{Label}` before the equation. You can then refer to the equation in other parts of your paper with `Equation\eqref{Label}`.

Input

```
If you are working with
\begin{equation}
\label{eqn1}
a^2 + b^2 = c^2
\end{equation}
you can refer to it as Equation \eqref{eqn1}
in other parts of the paper.
```

Output

If you are working with

$$a^2 + b^2 = c^2 \tag{8.2}$$

you can refer to it as Equation (8.2) in other parts of the paper.

If you want to discuss a system of equations (or inequalities), you will probably want to align them at the equal signs (or other signs). You can do this with the `align` environment. Each equation will need to have an ampersand (`&`) before the equal sign (or wherever else you choose to align the expressions) and each equation will need to end with \\. Each equation will get its own label; if you do not want an equation labeled you can put `\nonumber` before the end of the line. If you do not want any of the equations labeled you can use the `align*` environment.

Input

```
\begin{align}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align}
\begin{align*}
a + b &= c\\
    d &= e + f \nonumber\\
g + h &= i
\end{align*}
```

Output

$$a+b = c \quad (1)$$
$$d = e+f$$
$$g+h = i \quad (2)$$

$$a+b = c$$
$$d = e+f$$
$$g+h = i$$

The `align` environment can also be used to enter a string of equalities (or inequalities):

Input

```
\begin{align*}
a + b &= c + d\\
      &= e + f\\
      &= g
\end{align*}
```

Output

$$\begin{aligned} a+b &= c+d \\ &= e+f \\ &= g \end{aligned}$$

Matrices

You can create a matrix in math mode by using the `matrix` environment. The row elements of the matrix are separated by ampersands and each row ends with `\\`.

Input

```
\begin{equation*}
  \begin{matrix}
    1 & 2 \\
    345 & 6
  \end{matrix}
\end{equation*}
```

Output

$$\begin{matrix} 1 & 2 \\ 345 & 6 \end{matrix}$$

Notice a matrix does not come with delimiters; if you want a matrix with delimiters you can use the `pmatrix` environment to get parenthesis, `bmatrix` for brackets, and `vmatrix` for vertical lines.

Input

```
\begin{equation*}
\begin{pmatrix}
1 & 2\\
3 & 4
\end{pmatrix}
\begin{bmatrix}
a & b\\
```

`` c & d \end{bmatrix} \begin{vmatrix} A & B\\ C & D \end{vmatrix} \end{equation*} ```
Output
$$\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} \begin{bmatrix}a & b\\ c & d\end{bmatrix} \begin{vmatrix}A & B\\ C & D\end{vmatrix}$$

5 New commands

You can create your own commands by putting the line

`\newcommand{command}{definition}`

in the preamble of your document, where `command` consists of a backslash followed by letters and `definition` is what the command should be replaced with. For example, if you have

`\newcommand{\infd}{infinitely differentiable}`

in the preamble, then every time you enter `\infd` in your document the entire "infinitely differentiable" will be typeset.

Input
`Suppose the function $f$ is \infd.`
Output
Suppose the function f is infinitely differentiable.

You can create a command with arguments by putting

`\newcommand{command}[`n`]{definition}`

where n is the number of arguments, and `definition` can contain "#1", ..., "#n" to represent the n arguments. For example, if you have

```
\newcommand{\boldit}[1]{\textbf{\textit{#1}}}
```

in the preamble, then `\boldit{text}` will typeset `text` in bold italics.

Input
`Here is \textbf{bold}.` `Here is \textit{italics}.` `Here is \boldit{bold italics}.`
Output
Here is **bold**. Here is *italics*. Here is ***bold italics***.

For another example, if you add the following command

```
\newcommand{\vfrac}[2]{\ensuremath{\frac{#1}{#2}}}
```

in the preamble, then `\vfrac{x,y}` in the regular text mode will display a vertical fraction.

Input
`Let the desired fraction be \vfrac{x}{y}.`
Output
Let the desired fraction be $\frac{x}{y}$.

It is also possible to create your own environments, but it is unlikely you will need to do that for your MCM paper.

6 The header

While the header of a LaTeX document is configurable, a paper submitted to the MCM needs to have a header of the form

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That is, you need to print the team's control number in the upper left corner, and the page number of the current page with the total number of pages in the upper right corner. You can do this using the `fancyhdr` and `lastpage` packages; you will need to have

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
```

in the preamble. Also in the preamble, you can specify that you want the team number in the upper left of every page with the **\lhead** command:

```
\lhead{Team \# 3211}
```

You can put the appropriate page number in the upper right of every page with the **\rhead** command:

```
\rhead{Page \thepage{} of \pageref{Lastpage}}.
```

The command **\thepage** will output the page number, and **\pageref{Lastpage}** will output the number of the last page. You will also want to use **\cfoot{}** to prevent the page number from being at the bottom of every page. Putting

```
\usepackage{fancyhdr}
\pagestyle{fancy}
\usepackage{lastpage}
\lhead{Team \# 3211}
\rhead{Page \thepage{} of \pageref{LastPage}}
\cfoot{}
```

in the preamble will result in the appropriate

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---

at the top of the page and the line under the header. If you do not want this line to appear, you can add

```
\renewcommand{\headrulewidth}{0pt}
```

to the preamble.

7 Title and sections

LaTeX has a mechanism (i.e., \title{Title}, where Title is the actual title of the paper) for automatically inserting a title, however it is designed to insert more information than you want for an MCM paper. You should probably not use this mechanism, instead, put the title at the beginning of the paper by putting

```
\begin{center}
\Large  The Title of the Paper
\end{center}
```

right after `\begin{document}`.

Your paper will need to be divided into sections. To begin a section, you simply use the command `\section{section title}`. The section will be numbered automatically, and the section title will be in a larger, bolder, font.

Input

```
\section{Introduction}

Begin.

\section{Conclusion}

End.
```

Output

1 Introduction

Begin.

2 Conclusion

End.

Subsections can be created with the `\subsection{subsection title}` command.

Input

```
\section{Introduction}

\subsection{Restatement of the problem}

Problem.

\subsection{Random thoughts}

Thoughts.
```

```
\section{Conclusion}

End.
```

Output

1 Introduction

1.1 Restatement of the problem

Problem.

1.2 Random thoughts

Thoughts.

2 Conclusion

End.

You can similarly create subsubsections, but it is unlikely that you will need to use them.

You should create a table of contents for your paper. You can do this by inserting the command `\tableofcontents` where you want the contents to appear.

Input

```
\tableofcontents

\section{Introduction}

\subsection{Restatement of the problem}

Problem.

\subsection{Random thoughts}

Thoughts.

\section{Conclusion}
```

```
End.
```

Output

Contents

1 Introduction

1.1 Restatement of the problem

Problem.

1.2 Random thoughts

Thoughts.

2 Conclusion

End.

8 Footnotes

You can insert a footnote with the command

`\footnote{footnote text}`

Input

```
This is part of
a paper\footnote{This is a footnote.}
```

Output

This is part of a paper[a]

[a]This is a footnote.

Note that there should not be a space between **`\footnote`** and the previous word.

On a separate note, you must insert a space between the left parenthesis and the word preceding it; and a space between the right parenthesis and the word succeeding it.

9 Lists

LaTeX can automatically create various types of lists. A basic, bulleted list is created with the `itemize` environment. Each item in the list is preceded with an `\item`.

Input

```
\begin{itemize}
\item First item.
\item Second item.
\end{itemize}
```

Output

- First item.
- Second item.

A numbered list is created with the `enumerate` environment. Again, each item is preceded with an `\item` command.

Input

```
\begin{enumerate}
\item First item.
\item Second item.
\end{enumerate}
```

Output

1 First item.

2 Second item.

Lists can be nested; if enumerated lists are nested then the sublists are labeled with letters instead of numbers.

Input

```
\begin{enumerate}
```

```
\item First item.
      \begin{enumerate}
        \item First subitem.
        \item Second subitem.
      \end{enumerate}
\item Second item.
\end{enumerate}
```

Output

1 First item.
  (a) First subitem.
  (b) Second subitem.
2 Second item.

10 Figures

While LaTeX has some picture drawing capabilities, you will probably want to create graphics with another program and include them in your document. To do this, you will need to put the line

`\usepackage[pdftex]{graphicx}`

in the preamble. This will allow you to include pictures in the PNG, JPG or PDF formats. To do this, you use the line

`\includegraphics{filename}`

Since you may not want to worry about creating a picture that is already the correct size, you can scale the included graphic to the proper width using an optional argument: `\includegraphics[width=width]{filename}`.
The width can be specified in a variety of units, including `cm` and `in`.

Input

```
Here is a figure:

\includegraphics[width=2cm]{fig/figure.png}
```

```
Here it is again, smaller.

\includegraphics[width=1cm]{fig/figure.png}
```

Output

Here is a figure:

Here it is again, smaller.

To make the typesetting look better, you typically will not want the figure to appear right where you refer to it. A `figure` environment will tell LaTeX to place the figure in an appropriate spot and will allow you to give it a caption and a label.

```
\begin{figure}
\centering
\includegraphics[width=width]{figure}
\caption{Description}
\label{label}
\end{figure}
```

The `\centering` will make sure that the figure is centered between the margins, and LaTeX will place the figure at an appropriate place in the document. You can refer to it with

```
Figure \ref{label}
```

Input

```
The circle in Figure \ref{face} looks like a face.
\begin{figure}
\centering
\includegraphics[width=1.5cm]{figure.png}
\caption{This looks like a face.}
\label{face}
\end{figure}
```

Output

Figure 1: *This looks like a face.*

The circle in Figure 1 looks like a face.

11 Tables

Tables can be created in LaTeX with the `tabular` environment. The environment should look like

```
\begin{tabular}{format}
item & ... & last item \\
...
\end{tabular}
```

The `format` should consist of a letter `l`, `c` or `r` for each column;

- `l` for left justified columns;
- `c` for centered justified columns;
- `r` for right justified columns.

A vertical bar (`|`) in the format will create a vertical line between the appropriate columns. In each row, the row entries should be separated by an ampersand (`&`) and the row should end with a double backslash (`\\`). An `\hline` at the beginning of a row will put a horizontal line above the row in the typeset document.

Input

```
\begin{tabular}{|c|lr|}
\hline
Centered & Left & Right \\
\hline
0 & 1 & 2\\
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

Output

Centered	Left	Right
0	1	2
1	x	x^2

If you want an entry to extend across more than one row, for example if you want a title to apply to more than one row, you can use the following command:

```
\multirow{n}{format}{entry}
```

where the number n represents how many rows should be used, `format` is the formatting for a single row, and `entry` is the actual table entry. Likewise, you can also extend an entry across more than one column using the following command:

```
\multicolumn{n}{format}{entry}
```

Input

```
\begin{tabular}{|c|l|r|}
\hline
Centered & \multicolumn{2}{c|}{Spread out} \\
\hline
0 & 1 & 2\\
\hline
1 & $x$ & $x^2$\\
\hline
\end{tabular}
```

Output

Centered	Spread out	
0	1	2
1	x	x^2

As with figures, you typically will want a table to appear right when you refer to it. You can put it in a table environment, where you can give it a caption and label.

```
\begin{table}
```

```
\centering
\begin{tabular}{format}
...

\end{tabular}
\caption{Hello}
\label{label}
\end{table}
```

Input

```
The first four letters of the alphabet
are in Table \ref{alpha}.
\begin{table}
\centering
\begin{tabular}{c|c}
a & b \\
\hline
c & d
\end{tabular}
\caption{Some letters.}
\label{alpha}
\end{table}
```

Output

a	b
c	d

Table 1: *Some letters.*

The first four letters of the alphabet are in Table 1.

12 Bibliography

Your paper will need to have a comprehensive bibliography; it should contain all references that you've used. The simplest way to include a bibliography is to use the `thebibliography` environment, which begins with

```
\begin{thebibliography}{99}
```

and ends with

```
\end{thebibliography}
```

Each bibliography item begins with the command

```
\bibitem{label},
```

the label can be used to refer to it in the paper with the command

```
\cite{label}
```

Input

```
It was shown in \cite{Te} that termites eat wood.

\begin{thebibliography}{99}
\bibitem{Carp}
Carpenter, Bob. \textsl{The Life of Ants.}
Springer-Verlag, Berlin,
1994.

\bibitem{Te}
Terwilliger, Sam. \textsl{Termites.}
Prentice Hall, New York, 2004.
\end{thebibliography}
```

Output

It was shown in [2] that termites eat wood.

References

[1] Carpenter, Bob. *The Life of Ants.* Springer-Verlag, Berlin, 1994.

[2] Terwilliger, Sam. *Termites.* Prentice Hall, New York, 2004.

13 Sample file structure

The following is a sample skeleton of an MCM paper (You may modify or expand your sections according to the suggestions of MCM):

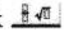
```
\documentclass[12pt]{article}

\usepackage[utf8]{fontenc}
\usepackage[doublespacing]{setspace}
\usepackage{textcomp}
\usepackage{amsmath,amssymb}
\usepackage{fancyhdr}
\usepackage{lastpage}

\pagestyle{fancy}
\lhead{Team \# control number}
\rhead{Page \thepage{} of \pageref{Lastpage}}
\cfoot{}

\begin{document}

\begin{center}
{\Large Title}  Title of the Paper
\end{center}

\tableofcontents

\section{Introduction}

A restatement of the context problem, a survey of existing
results, and a brief description of your approach to the problem

\section{Assumptions and Justifications}

A list of your assumptions and justifications

\section{Model Design}

A description of your model

\section{Conclusions}
```

```
A description of model testing, your results, and the strengths
and weaknesses of your approach

\begin{thebibliography}{99}

A list of references

\end{thebibliography}

\end{document}
```

Chapter 9 MathType

MathType is an equation editor for typesetting math in MS Word documents, powerpoint slides, and other applications. Unlike LaTeX that needs compiling, MathType is a WYSIWYG application. MathType refers to mathematical expressions as equations. We will use equations and expressions interchangeably. You can download MathType from its official web site at `http://www.dessci.com/en/products/mathtype/`.

You may enter equations using the following two methods.

1. Open MathType and enter the desired equations in the input area provided by MathType. Then copy the equations to the desired locations in the Word document. MathType provides a window with an input area for entering equations (see Fig. 9.1). The built-in modules provide common math templates. When entering an equation, you would need your keyboard to type in characters and select appropriate templates from the built-in modules. For example, suppose that you want to enter a fraction. Click  (for fractions and roots) to enter the following module (see Fig. 9.2).

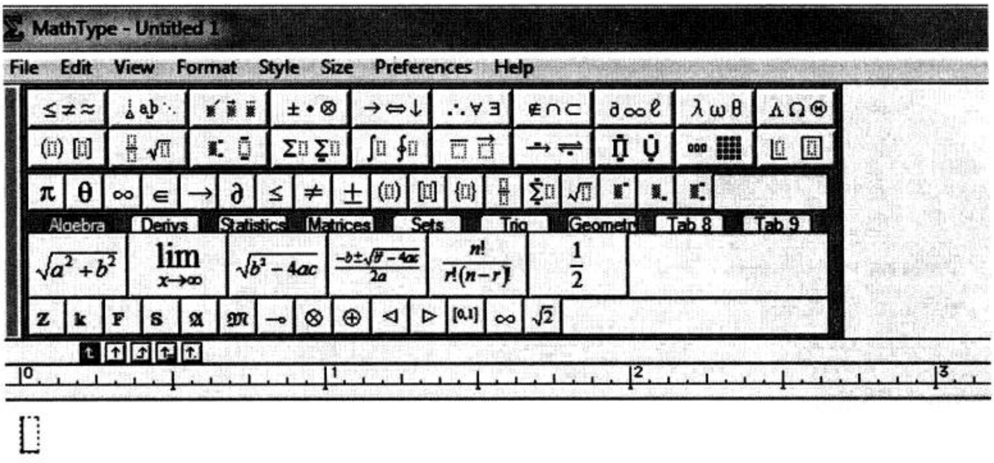


Fig. 9.1 MathType window

Then select a particular template based on your need, and enter variables or numbers to complete the fraction.

Fig. 9.2 Fractions and roots

2. Use MathType inside your Word document. That is, on the tabbed bar of the Word window, click `MathType`. At the location of the document where you want to enter your equation, select one of the following displaying options: `Inline`, `Display`, `Left-numbered` and `Right-numbered` (see Fig. 9.3) Clicking on one of these options will pop up a MathType window for you to enter equations. When you are done entering, exit MathType and you will see that the equations you just created are included in your word document at the location you selected.

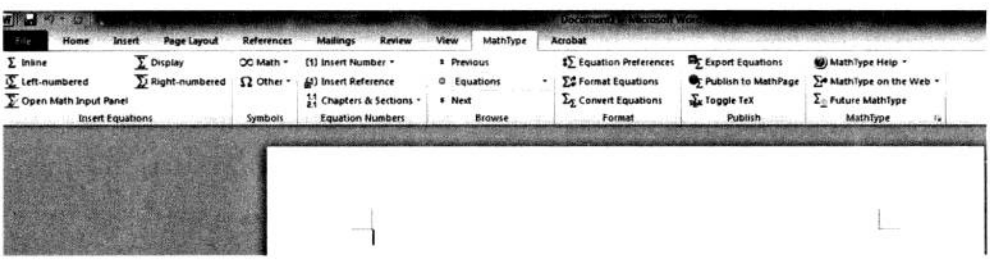


Fig. 9.3 Word window with MathType

We will use examples to illustrate how to typeset math using MathType.

1 Fractions and square root

Suppose that you want to create the following equation:

$$y = \sqrt{\frac{3}{16}} \sin x - c^2 \pm \mu \tan x.$$

Follow these steps:

1. Type "$y =$" in the typing area of the MathType window. Click . Click to select the square root template, and then click to select the fraction

template. Type "3" in the numerator and "16" in the denominator to get

$$y = \sqrt{\frac{3}{16}}$$

2. Type "$\sin x-$" outside of the square root. MathType automatically identifies function sin and displays it appropriately.
3. To enter the superscript for squaring, first type a letter "c", then click and select an appropriate superscript template.
4. On the tabbed bar of the MathType window, select the plus and minus sign $\pm$ at $\lfloor\pm\rfloor$.
5. Click $\underline{\lambda\omega\theta}\rfloor$ (the Greek symbol module), and select letter μ. Then type "$\tan x$". You now have the desired equation as follows, see Fig. 9.4.

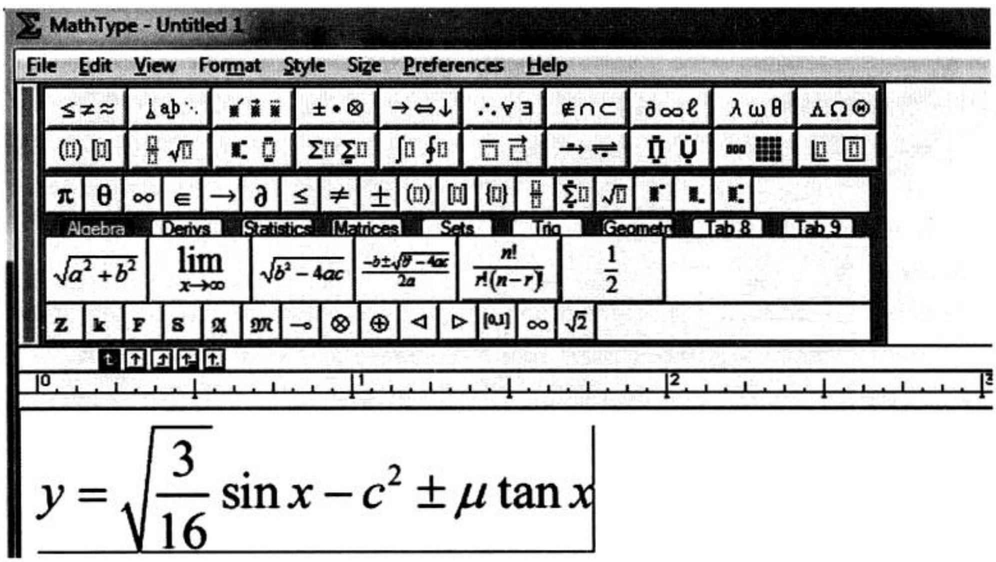


Fig. 9.4 Type fractions and square root

2 Integrals, subscripts, and summations

Suppose that you want to typeset the following expression:

$$\sum_{i=1}^{\infty}\frac{1}{i} \quad \iiint_E \mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

Follow these steps:

1. Click to enter the summation module and select an appropriate summation template.
2. Insert `Fraction` $\frac{1}{i}$.
3. Click to enter the integral module and select an appropriate integral template.
4. Type dxdydz.

3 Multiple equations

Suppose that you want to typeset the following multiple equations:

$$\sigma^2 = \begin{cases} \dfrac{1}{3}\displaystyle\sum_{i=1}^{n} x_i^2 & i \geq 2 \\ 2 & i = 1 \end{cases}$$

Follow these steps:

1. Click to enter the Greek-symbol module and select "σ". Then click to enter the superscript and subscript module, select the desired superscript template, and type "2"。
2. Type "="。
3. Click to enter the delimeter module, and then click to select left brace. Press the enter key to generate two input boxes, one in each row. The left brace will automatically expand vertically to cover these two rows. (Note that you may also press the enter key at the end of the current expression to create a new row.)
4. In the first input box enter the desired fraction and summation. Click to enter the module of speical symbols, and then click a few times to create sufficiently large blank space. Enter "$i \geq 2$". (Note that MathType does not allow you to use the space bar to enter blank space.)
5. In the second input box enter "2", blank space (in the same way as in the first input box), and "$i = 1$".

   In the MathType typing area you will now see Fig. 9.5.

4 Matrices and determinants

On the tabbed bar of the MathType window you can find the `Matrices` option, which provides five templates for entering matrices. However, there is no option

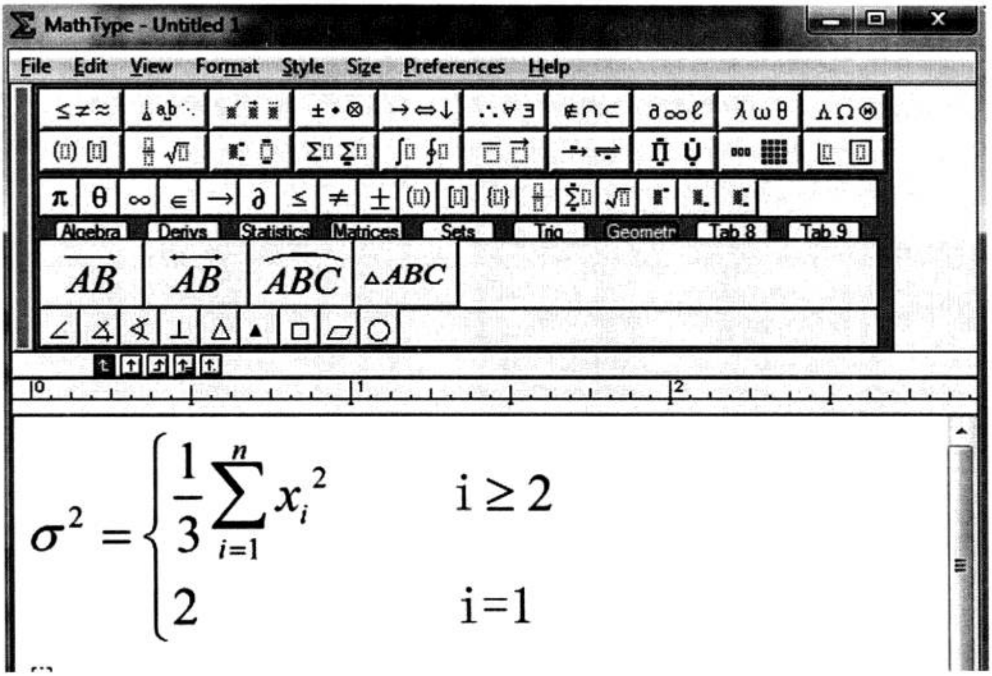


Fig. 9.5 Typeset multiple equations

for entering determinants on the tabbed bar. Follow the following steps to enter a determinant.

1. Click  to enter the delimeter module. Then click  to select the delimiters for determinants.
2. Click  to enter the matrix module, from which select a suitable template to enter the needed determinant.

5 Saving user-created expressions in the tabbed bar

MathType provides a special tabbed bar (shown in Fig. 9.6) for typesetting equations in common math subjects, including algebra, statistics, integrals, matrices, sets, trigonometry, and geometry.

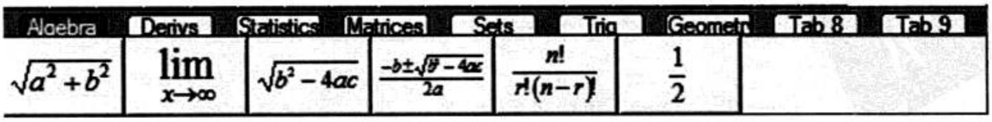


Fig. 9.6 MathType tab bar

MathType supports all math symbols commonly used in the literature and allows users to personalize expressions. It also allows users to move the expressions they created to the tabbed bar for easy access later. Follow these steps to do so:

1. Select an expression that you want to move to the tabbed bar. For example, select the following expression (see Fig. 9.7).

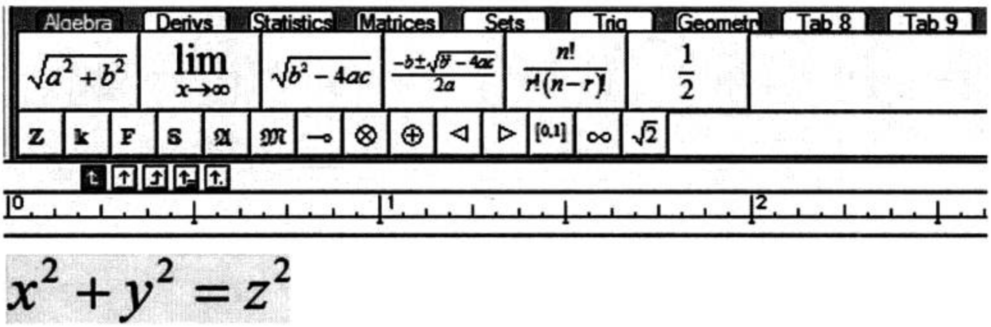


Fig. 9.7 Select an expression

2. Hold the left button of your mouse and drag it into the black rectangle position as shown in Fig. 9.8.

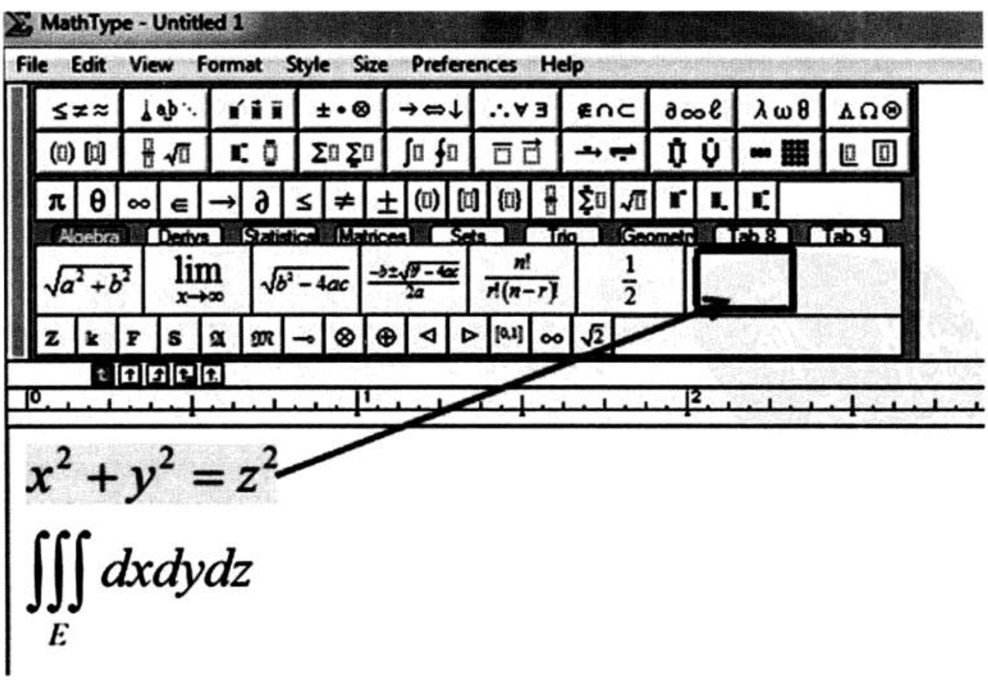


Fig. 9.8 Drag the expression into the black rectangle

Your expression will now appear in the tabbed bar, indicating that you have successfully added your expression. If you know which existing category this expression belongs to, you could put it under that category. Otherwise, you could simply move it to the tabbed bar under Tab 8 or Tab 9 as a self-created expression, as shown in Fig. 9.9.

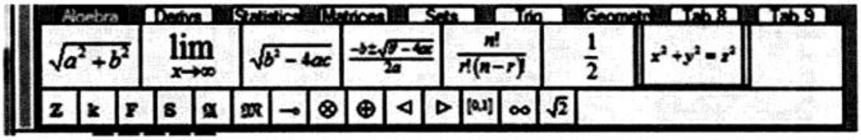


Fig. 9.9 Add your expression under Tab 8 or Tab 9

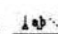
This method also applies to Greek symbols, or any other symbols in the `Symbol` palettes and the `Template` palettes.

To remove an expression from the tabbed bar, hold the right button of your mouse, and select `delete`.

6 Alignment

Suppose that we would like to create the following two expressions aligned at $\leq$:

$$\int_0^1 a(x)\,dx \leq \limsup_{n\to\infty} \phi_n(a),$$
$$\int_0^1 a(x)b(x)\,dx \leq \limsup_{n\to\infty} \varphi_n(a,b).$$

1. For the first equation, enter the desired integral on the left-hand side of the $\leq$ sign. Enter appropriate blank space between $a(x)$ and dx using the  module.
2. Complete the rest of the equation and press the `Enter` key.
3. Copy and paste the first equation and then modify it to create the second equation. (This will save time.) You will now see that the two equations are aligned at the left-hand side as follows:

$$\int_0^1 a(x)\,dx \leq \limsup_{n\to\infty} \phi_n(a)$$
$$\int_0^1 a(x)b(x)\,dx \leq \limsup_{n\to\infty} \varphi_n(a,b)$$

4. To align them at $\leq$, click `Format` in the menu see Fig. 9.10, and then click

`Align at =` to get the desired alignment, shown in Fig. 9.11.

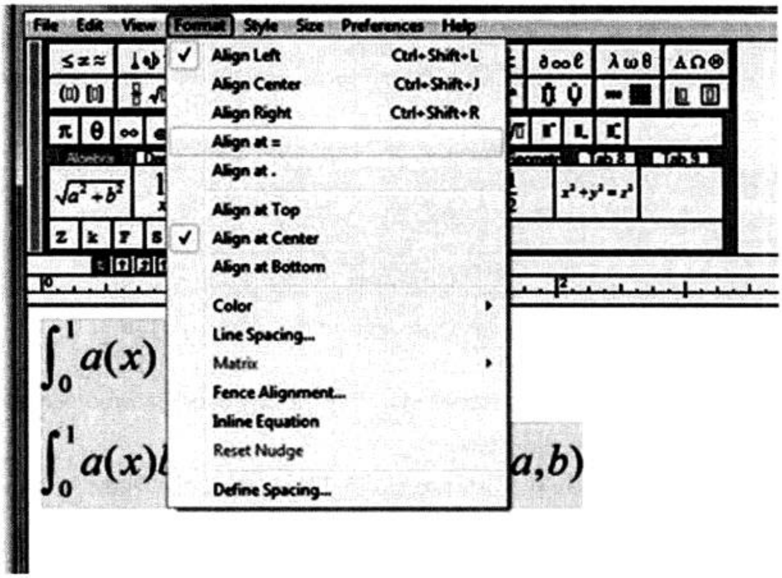


Fig. 9.10 Format menu

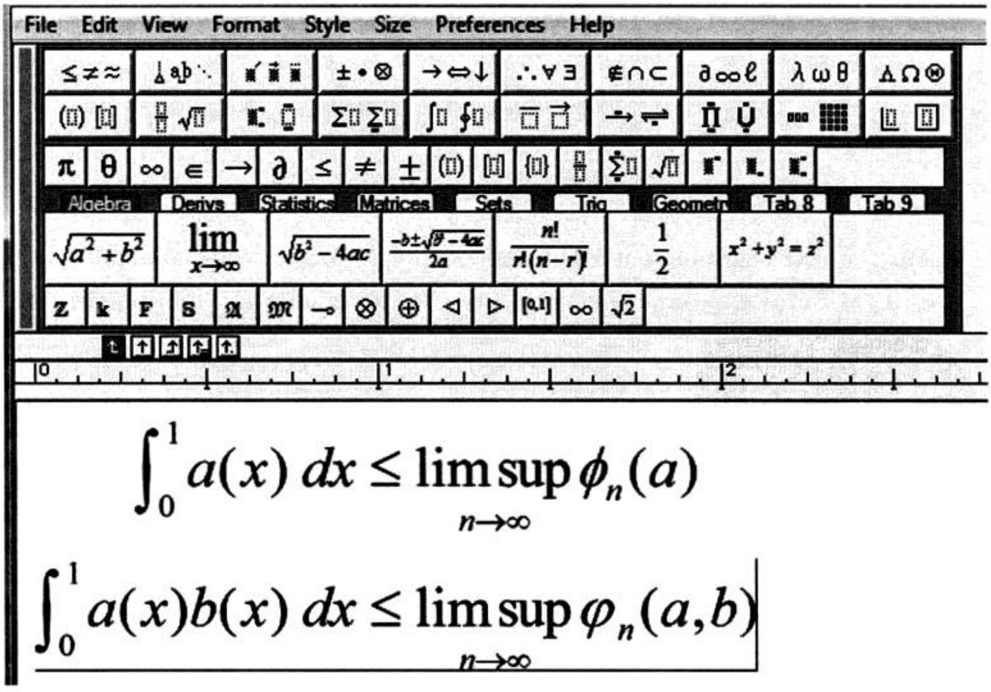


Fig. 9.11 Get the desired alignment

7 Change fonts and styles

Suppose that we want to change the following equation from the default `Math` style to the `Euclid` style:

$$u = \phi \cdot exp\{\frac{1}{2}\sigma(x+y)\}.$$

1. On the status bar at the bottom of the typing area of the MathType window you should be able to see `Style:Math`. If not, click `Style` on the menu and then click `Math`.
2. Create the equation. Click `Style` and then `Define` to pop up the following dialog (see Fig. 9.12).

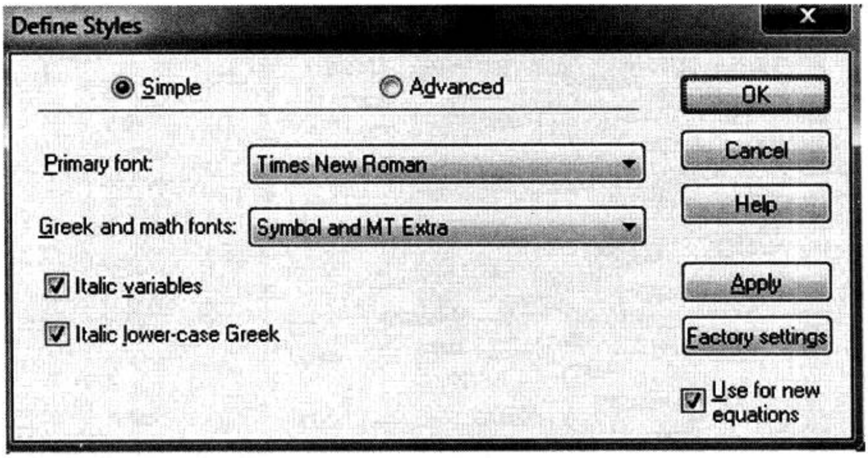


Fig. 9.12 Define Styles dialog

3. Change `Primary font` to `Euclid` and change `Greek and math fonts` to `Euclid Symbol and Euclid Extra`.

   Note that you may reset the settings by selecting `Factory settings`. Click `Advanced` for more options, see Fig. 9.13.

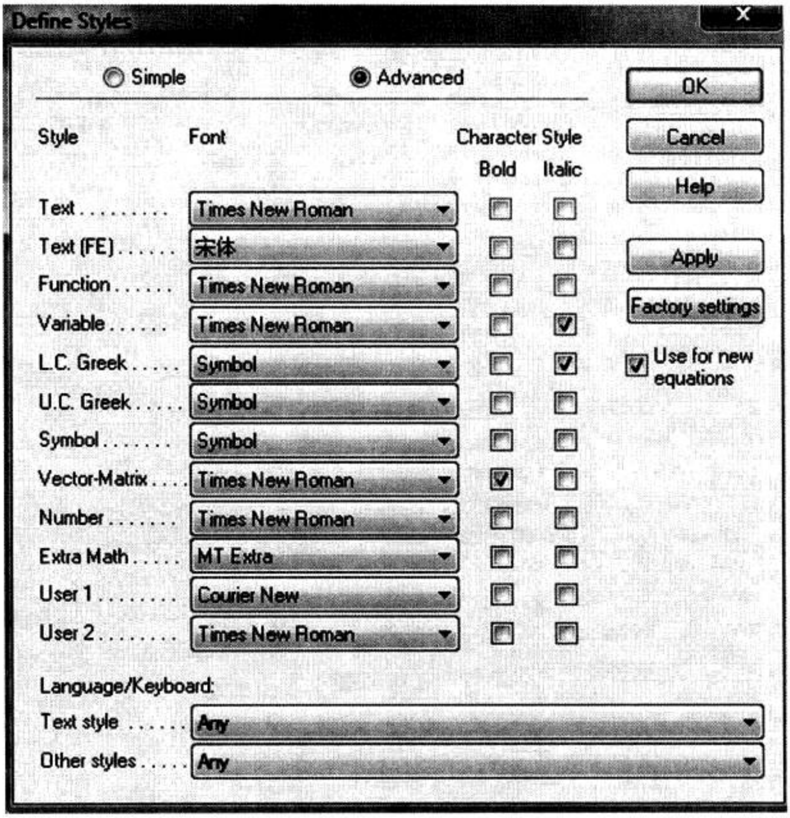


Fig. 9.13 Advanced menu

You may set `Text`, `Function`, and `Symbol` with different styles according to your needs.

8 Numbering equations

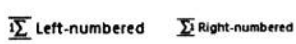
Suppose that we want to number the following two equations as "(1.1)" and "(1.2)":

$$\cos^2\theta + \sin^2\theta = 1 \tag{1.1}$$

$$\cos^2\theta - \sin^2\theta = \cos(2\theta) \tag{1.2}$$

Follow these steps:

1. Open your Word document and click the MathType option. You can find the following



on the tabbed bar, which provides two methods to number equations. The first method places a numbering on the left-hand side of an equation, and the second places that on the right-hand side.

2. Click Σ Right-numbered to automatically generate numberings according to which chapter and which section the equations are located, and a MathType window will pup up. Create the first equation in the Mathtype typing area. Exit Mathtype and you will see the numbering "(1.1)" at the right-hand side of the equation in the Word document, where the section number appears before the expression number, separated by a dot.
3. Create and number the second equation in the same way. You'll see the following in your Word document:

We have equation:

$$\cos^2\theta+\sin^2\theta=1 \qquad (1.1)$$

$$\cos^2\theta-\sin^2\theta=\cos(2\theta) \qquad (1.2)$$

MathType automatically numbers each equation in the correct order. Should you want to change the numbering of a certain equation, click on Chapters & Sections ▾, and then click `Modify Break` to get the dialog, shown in Fig. 9.14.

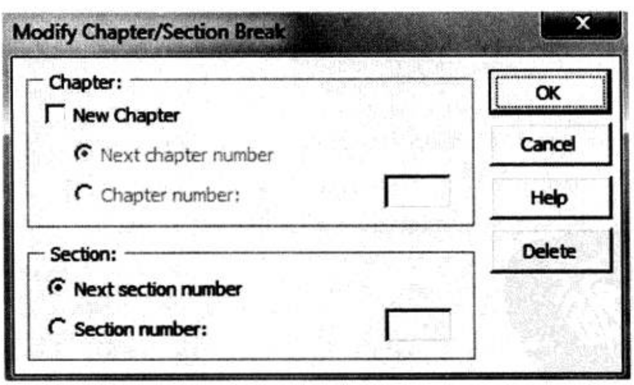


Fig. 9.14 Modify Chapter/Section Break dialog

Enter the desired numbering there.

9 Word document settings

When creating a Word document containing equations, you probably want them to match with the font and size of the text body, and maintain a consistent format.

While Word and MathType allow you to change fonts and sizes directly, it is better to define a new style to specify them, for doing so will make it easy to modify the format of your equations and text. This is done by changing the definition of style, e.g., from "Arial" to "Times New Roman", or from "10 pt italic" to "12pt plain". After the new style is defined, your document will be reformatted automatically with the new setting.

As an example, let us consider how to produce a document with "12pt Times New Roman". Define a MathType setting to match with the Word document as follows:

1. In MathType, select `Style`. Then select `Define` and choose `Times New Roman` as the primary font. Font `Use for new equations`.
2. Repeat the above steps. Then choose `12 pt` and select `Use for new equations`.
3. Back to Word, and select the MathType option, from there select `Equation Preferences` and choose `MathType's 'New Equation' preferences`. This new setting will automatically apply to any new equation.
4. Now change the style of the text body. Click `Home`. In the `Styles` box, choose a style and right click your mouse, and then choose `Modify`. Find `Formatting`, and select `Times New Roman` and `12`.

10 Default fonts and styles

The font chosen to typeset characters and symbols for the main body of the text is referred to as the primary font. The default primary font is "Times New Roman". Equations should use the primary font or different forms of the primary font. For instance, the logarithmic functions (log, ln, lg), the trigonometry functions (sin, cos, tan), the modular operation (mod), and the minimum and maximum operations (min, max) should use the primary font. Variables should use the primary font italic. Vectors and matrices should use the primary font italic and bold.

Listed below are the common styles in MathType:

Math The `Math` style is the default style for characters and symbols in mathematical expressions.

Text The `Text` style is the default style for entering text.

Function The `Function` style is used to display the logarithmic functions, the trigonometry functions, the modular operation, the minimum and maximum operations, and other functions and operations with a common fixed name.

Variable The `Variable` style is used to display characters in mathematical expressions.

Greek-Symbol The `Greek-Symbol` style is used to display Greek characters. It also applies to mathematical operators.

Vector-Matrix The `Vector-Matrix` style is used to display vectors and matrices. It is typically the same as the Variable style but in bold.

User 1 and User 2 These two tabs are for users to define their own styles.

MathType automatically assigns certain styles and sizes to certain types of characters with appropriate spacing, based on function identification and character substitution.

Under the styles of `Math`, `Variable`, `Function`, `Vector-Matrix`, and `Greek-Symbol`, MathType may automatically substitute certain characters appropriately. For example, the "minus" sign that is entered as a hyphen under the `Text` style will be replaced with a real "minus" sign, which is longer than a hyphen.

MathType automatically assigns an appropriate size for characters and symbols in equations, selected from the following types: full, subscript (superscript), two-layer subscript (superscript), symbol, and sub-symbol.

11 Tips

In addition to the default settings, MathType also allows the user to define their own styles, but for writing an MCM paper, the default settings would be sufficient. To use MathType more efficiently, we provide a few tips in this section.

Modify equations in the document

To fix a typo in an existing expression or make a minor amendment, double click on the expression you want to modify. The MathType interface will appear with the selected expression. Perform the modification you desire, save it, and close MathType. The modification will appear in the Word document.

Change the font and size for multiple expressions at the same time

You may need to change the font and size for the entire paper. Follow these steps:

1. Open the Word document, double click an expression. In the MathType popup window, click `Size` then `Define`, and change the size according to your need.
2. Click `Preference`, `Equation preference`, and `Save to file` in order. Then save this preference to a file with a file name different from the default name. Close MathType and go back to Word.
3. Select the MathType option on the Word tabbed bar. Click `Format equation`, `MathType preference File`, and `Browse` in order. Then click the file you just saved, select `Whole document`, and press `Ok`.

More symbols

MathType offers thousands of symbols for typesetting mathematical expressions. You may find them from the tabbed bar, the `Symbol` option, and the `Template` option. If you still cannot find what you need, click `Edit` and then `Insert Symbol` to popup a window with more symbols. For your convenience, we recommend that you save commonly used symbols in the tabbed bar.

Use existing standard expressions

MathType has a category-breakdown tabbed bar, where you can find templates existing standard equations in the subject area. For example, the formula of roots for quadratic equations is saved under `Algebra`, and standardized random variables are saved under `Statistics`.

Change font, color, and size

Font, color, and size can be changed by clicking, respectively, `Style` then `Define`, `Format` then `Color`, and `Size` then `Define`.

Save favorite equations

Drag the selected expression into the blank area on the large tabbed bar or the small tabbed bar to save it. When it is no longer needed, right click on it and use the delete option to delete it.

Shortcut keys

Most commands in MathType can be executed using shortcut keys to save time. It is useful to memorize a few shortcut keys for the items you would use often.

Listed below are common keystrokes in the categories of mathematical operators, Greek symbols, and formatting.

Math operators

Fraction

Ctrl+F (regular fraction; i.e., hold the Ctrl key while pressing F)
Ctrl+/ (forward slash fraction)

Square root

Ctrl+R (square root)
Ctrl+T-N (root of high dimension; Ctrl-T-N means to press Ctrl+T, release both keys, then press the N key)

Subscript and superscript

Ctrl+H (superscript)
Ctrl+L (subscript)
Ctrl+J (superscript and subscript)

Inequalities

Ctrl+K-, ($\leq$)
Ctrl+K-. ($\geq$)

Differentials and integrals

Ctrl+Alt+' (first derivative)
Ctrl+Shift+" (second derivative)
Ctrl+I (definite integral)
Ctrl+Shift+I (indefinite integral)

Brackets

Ctrl+9 or Ctrl+0 (parentheses)
Ctrl+[or Ctrl+] (brackets)
Ctrl+{ or Ctrl+} (curly brackets)

Others

Ctrl+Shift+- (bar)
Ctrl+Alt+- (vector arrow)

Greek Symbols

Press Ctrl+G, release both keys, then enter a lower-case letter. You'll get the relevant lower-case Greek symbol.
Press Ctrl+G, release both keys, then enter an upper-case letter. You'll get the relevant upper-case Greek symbol.

Formatting

Change visual size These shortcut keys only change visual size, which do not affect the real size.

Ctrl+1 (100%)
Ctrl+2 (200%)
Ctrl+4 (400%)
Ctrl+8 (800%)

Tweak and spacing

Ctrl+Arrow (select an item you want to move, then press Ctrl+Arrow to move the item in the direction of the arrow)
Ctrl+Alt+Space (insert space between characters)

Define your own shortcut keys

You may define your own shortcut keys for a specific symbol or equation as follows:

1. In the MathType window, right click on a symbol (e.g., π), then click `Properties` to pop up the following window, see Fig. 9.15.

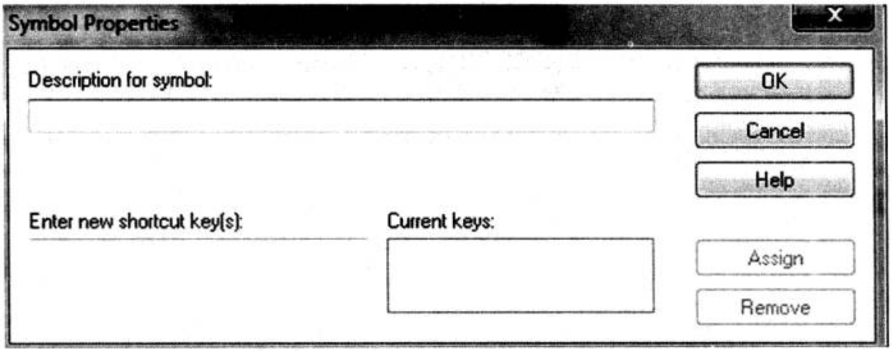


Fig. 9.15 Symbol Properties dialog

2. In the `Enter new shortcut key(s):` box, type the shortcut key you selected, and click `OK`.

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