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Letters to the Editor

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Graphic design of pinhole cameras

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Recently while looking for information on pinhole cameras, we were pleased to come across Young's paper "Pinhole Optics".[1] We have been using a graphic technique to analyze and optimize the pinhole size and focal length using the same factors as those discussed by Young and other authors.[2] The graphic technique is based on the use of the transfer function (TF) of optical elements described by Scott[3] to construct the transfer function of a circular pinhole camera. The transfer function is the response of a component or system to a pattern of lines having a sinusoidally varying radiance at varying spatial frequencies. The two transfer functions used in constructing the circular pinhole camera are the same as the two elements described in Ref. 1 as factors limiting resolution:

(a) The diffraction effect of Ref. 1 is described by the transfer function of a diffraction limited circular lens as shown in Fig. 1. The equation for the transfer function of the lens is[4]

$$\mathrm{TF}_{(\mathrm{lens})}(f) = \frac{2}{\pi}\{\cos^{-1}(f/f_c) - (f/f_c)[1-(f/f_c)^2]^{1/2}\}, \tag{1}$$

where f is spatial frequency in cycles per radian (cycles/rad), and $f_c = d/\lambda$ is the cutoff frequency. d is the diameter of the pinhole, and λ is the wavelength of light.

(b) The projection of the pinhole on the image plane (also noted in Ref. 1) is described by the transfer function of a circular aperture or field stop as shown in Fig. 2. The equation for the transfer frunction of the aperture is

$$\mathrm{TF}_{(\mathrm{apert.})}(f) = 2J_1(\pi d_p f)/d_p f\pi, \tag{2}$$

where J_1 is the Bessel function of order one, and d_p is the projected pinhole angular diameter at the image plane.

The complete transfer function for a circular pinhole camera system has been derived by Swing and Rooney[6] to be of the following form:

$$\mathrm{TF}_{\substack{\text{pinhole}\\\text{camera}}}(\sigma) = F_e(\sigma)\,\mathrm{Bessinc}[\pi d(1+Z_2/Z_1)\sigma F_e(\sigma)], \tag{3}$$

where σ is the spatial frequency in cycles per distance, Z_1 and Z_2 are, respectively, the object and image distances measured from the pinhole position, the function Bessinc (σ) is equal to $2J_1(\sigma)/\sigma$, and $F_e(\sigma)$ is identical to the transfer function of the lens as given by Eq. (1). Since the quantity $d(+Z_2/Z_1)$ is equal to the projected pinhole diameter on the image plane, Eq. (3) can be put into a simple form with the substitution of Eq. (1) and (2) as follows:

$$\mathrm{TF}_{\substack{\text{pinhole}\\\text{camera}}}(f) = \mathrm{TF}_{(\mathrm{lens})}(f) \times \mathrm{TF}_{(\mathrm{apert.})}[f \times \mathrm{TF}_{(\mathrm{lens})}(f)]. \tag{4}$$

Graphical solutions to Eq. (4) can be accomplished with the following procedure. Using the same log–log paper the transfer function curves were plotted on, establish the hori-

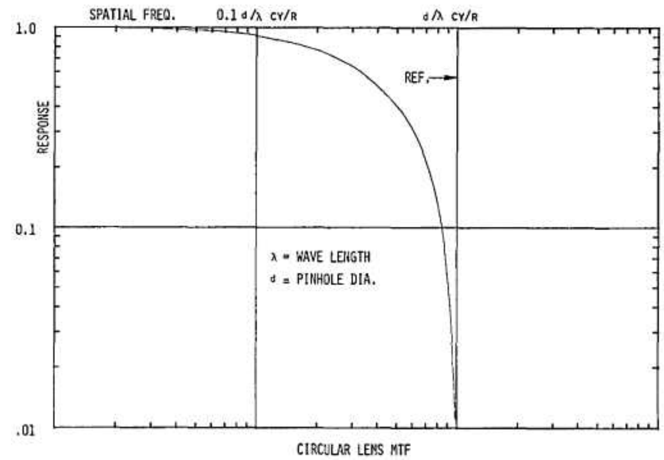

Fig. 1. MTF for a diffraction limited circular lens plotted on a log–log scale. Spatial frequency is in units of cycles per radian with the cutoff frequency as a reference.

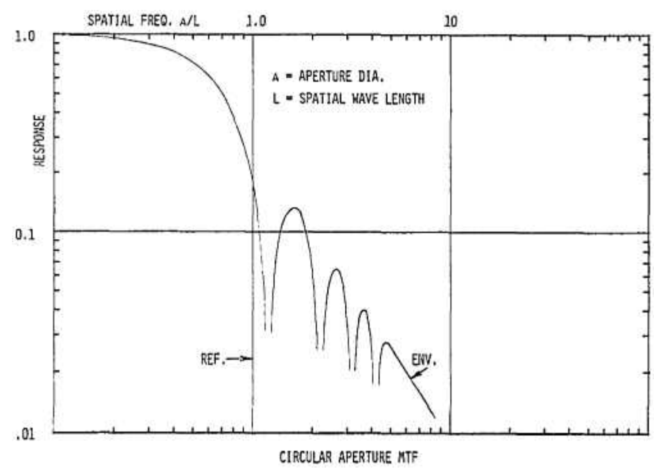

Fig. 2. MTF for a circular aperture plotted on same log–log scale as in Fig. 1. The spatial frequency is normalized to aperture diameter.

zontal scales: spatial frequency in cycles/rad increasing to the right. Using the templates, trace curves on the graph (Fig. 3) as follows: Assuming that the angular diameter of the pinhole's projection onto the image plane is 5 mrad, locate the aperture template (from Fig. 2) with its reference at 200 cycles/rad or 5 mrad (curve A). Note that this angle is the diameter of the pinhole divided by the focal length of the camera, and thus for distance objects, the F/number (F = focal length) is numerically the same as the reference frequency or F/200. Assuming $\lambda = 0.0005$ mm, pinhole diameter = 0.4 mm, and $\lambda/d = 1.25$ mrad, locate the reference on the lens template

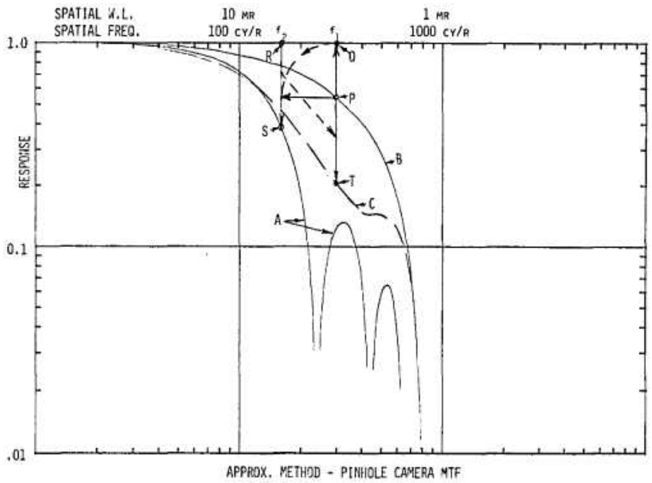

Fig. 3. Graphic solution for the MTF of a pinhole camera with pinhole diameter of 0.4 mm, camera F/number of 200, and $\lambda = 0.5\ \mu$m. See text for details on various steps.

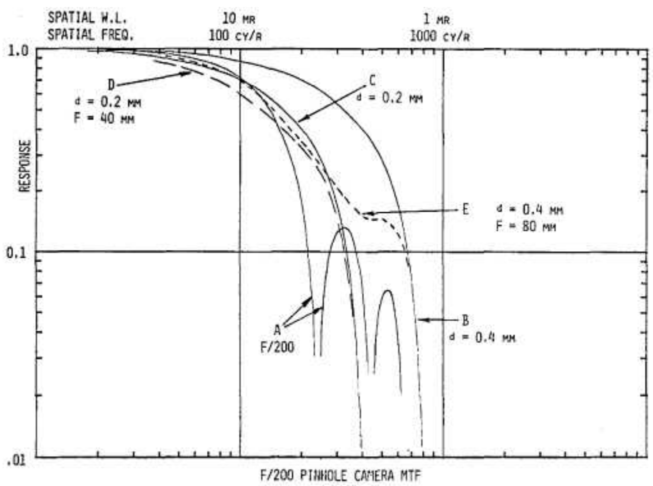

Fig. 4. Graphic design of an F/200 pinhole camera with pinhole diameters of 0.2 mm (curve D) and 0.4 mm (curve E).

(from Fig. 1) at 1.25 mrad (800 cycles/rad) and trace curve B. To determine the camera transfer function, the next four steps are illustrated in Fig. 3: (1) at any frequency f_1, measure the ordinate PQ from the lens response curve; (2) determine f_2 with $QR = PQ$; (3) measure the ordinate RS from the aperture response curve at f_2; and (4) subtract this ordinate RS from the ordinate of the lens response curve at f_1 to determine the point T, where $PT = RS$. The resulting curve C generated by the above procedure is a graphic solution to Eq. (4).

Figure 4 shows an F/200 camera with pinhole diameters of 0.2 mm and 0.4 mm. The camera curve on the left (curve D) shows 40% response to 160 cycles/rad with a 40-mm focal length and no spurious resolution (reversed contrast). With a 0.4-mm pinhole (curve E) there is slight improvement at 40% response, but at 10% response (low contrast), resolution is improved from 320 cycles/rad to 640 cycles/rad. If the F/number is kept constant, a smaller (than 0.2-mm) pinhole will reduce resolution rapidly, while a larger (than 0.4-mm) pinhole will introduce spurious resolution. It appears, therefore, that the optimized pinhole camera at any F/number will fall between the values $F = 0.5d^2/\lambda$ (curve D) and $F = 0.25d^2/\lambda$ (curve E). All the values seen by the authors in pinhole camera literature fall between these two values.[7] Resolution at 10% response is approximately $0.75d/\lambda$ cycles/rad or $0.75d/\lambda F$ lp/mm.

The use of the aperture curve as illustrated applies to distant objects. For close objects the projected aperture at the image plane is larger than the pinhole; its angular size from the center of the pinhole must be calculated to locate the template reference. For instance, when object distance equals image distance the size of the projected aperture is twice the pinhole size, and the aperture curve must be shifted to half the frequency indicated by the pinhole size. Since the F/number remains the same, it will be numerically twice the new aperture reference frequency.

Any desired combination of pinhole and focal length can be tried quickly and easily by this method. The results provide a quantitative view of response or contrast at varying spatial frequencies and thus a meaningful measure of resolution.

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Improved microimages from in-line absorption holograms

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In-line holography has become an established tool in particle and aerosol analysis, and reviews of the various applications of the technique are now available.[1–4] However, although the technique has many important applications, its usefulness has been limited by the quality of the reconstructed image. Considerable background noise has been observed in the images,[5] and this has been attributed to the presence of the virtual image and, at high concentrations, to the out-of-focus particles. However, little consideration has been given to the developing process, and its importance in the production of high quality microimages has rarely been recognized. In the following account the desired characteristics of microimages for subsequent quantitative evaluation will be discussed, and the processing techniques and materials needed to produce such results will be described. The use of these methods should usefully extend the range of applications of this type of holography.

High contrast images are desirable in in-line holography, particularly for particle size analysis, since each image must be clearly distinguished from the bright background characteristic of this type of holography. Filtering techniques have been used in an attempt to reduce the background noise,[6–8] but this adversely affects the image quality. In order to ob-