Quantum Spin Hall Effect

B. Andrei Bernevig and Shou-Cheng Zhang

PRL **96,** 106802 (2006)

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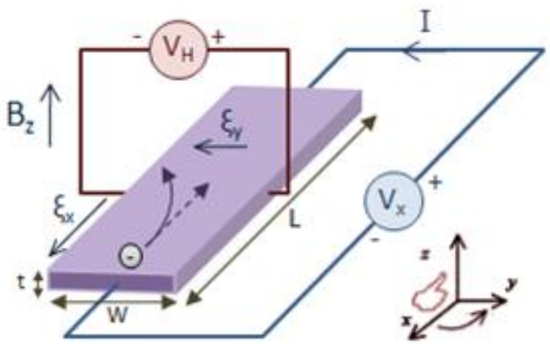
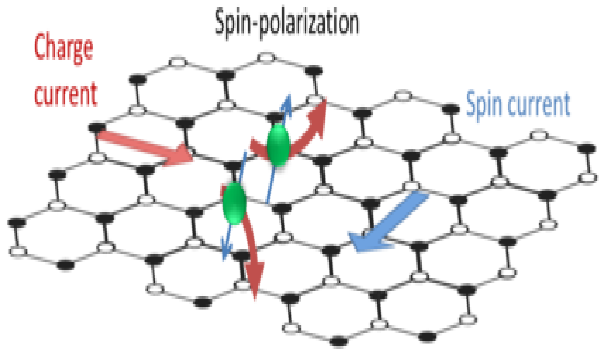
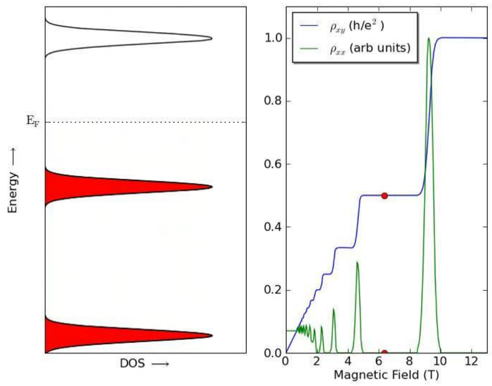
(Part of) Members of the “Hall effect family”



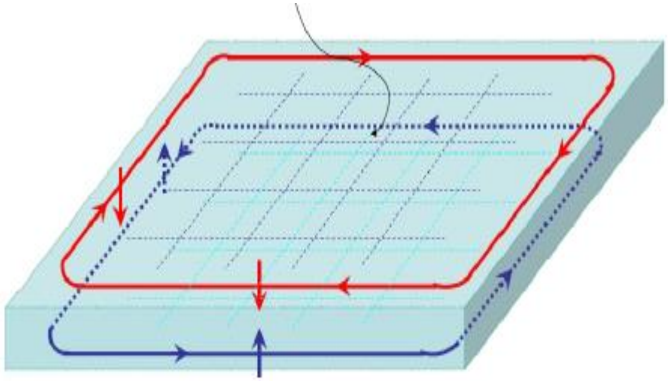
Hall Effect



Spin Hall Effect



Quantum Hall Effect

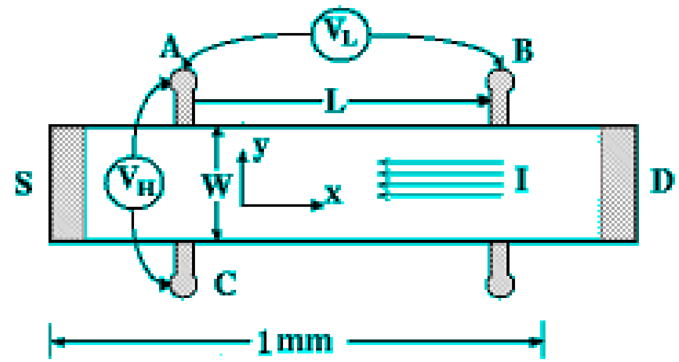


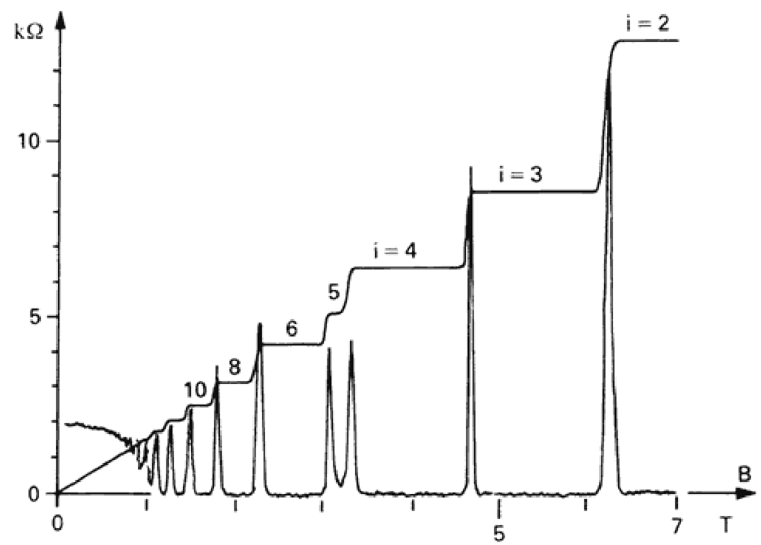
Quantum Spin Hall Effect

First three figures are from Wikipedia; the fourth is from PRL 96, 106802 (2006).

Quantum Hall Effect

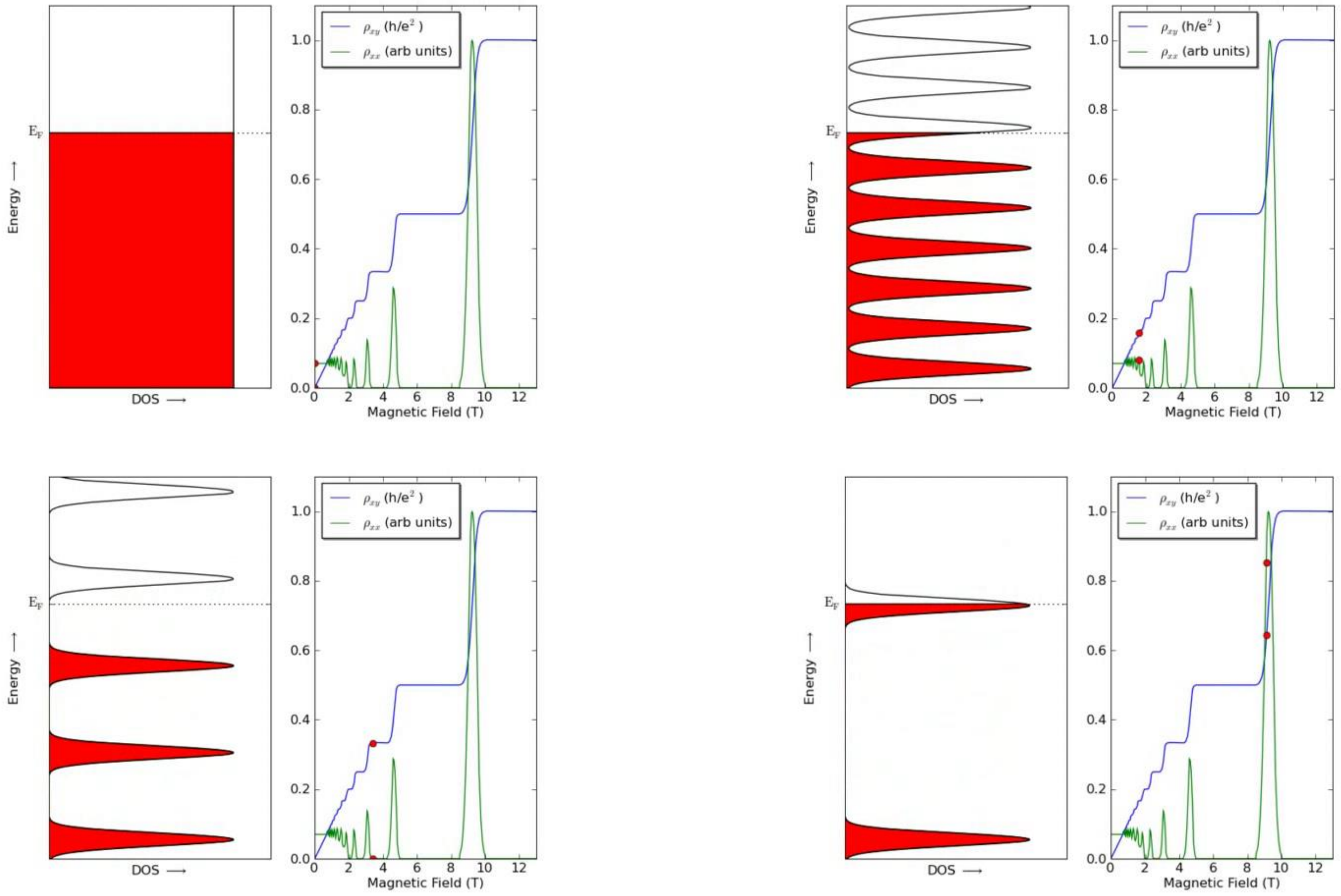
von Klitzing





Figures from PRL45, 494 (1980)

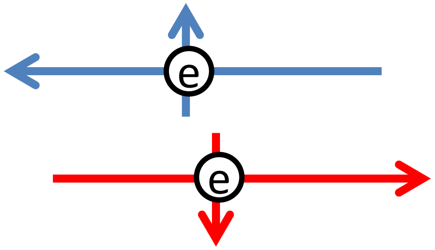
Explanation to QH: Landau Level



Figures from Wikipedia - Quantum Hall Effect

Spin Hall effect

Spin Hall effect is the spin counterpart of charge Hall effect



pure spin current

spin Hall effect

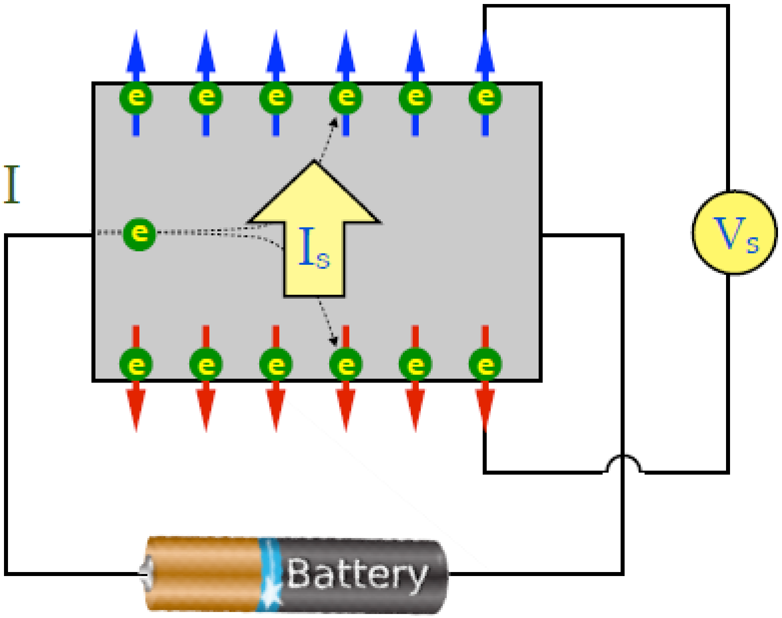
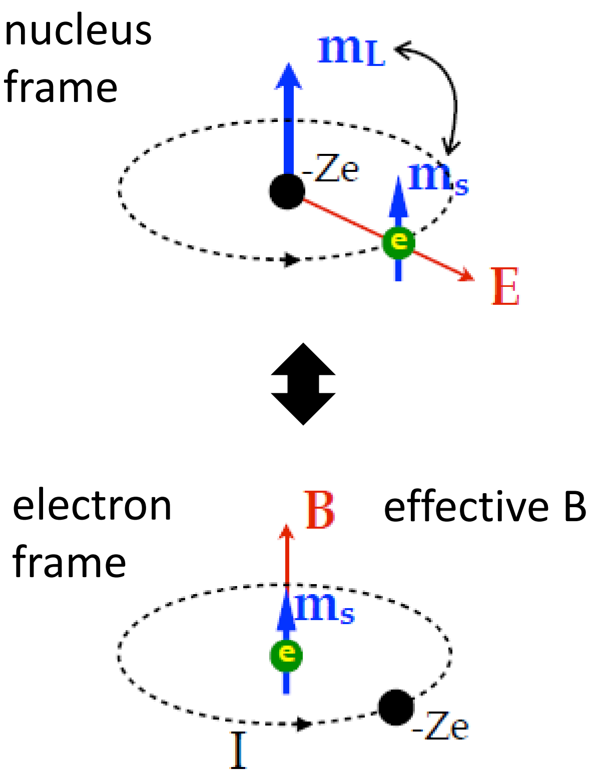
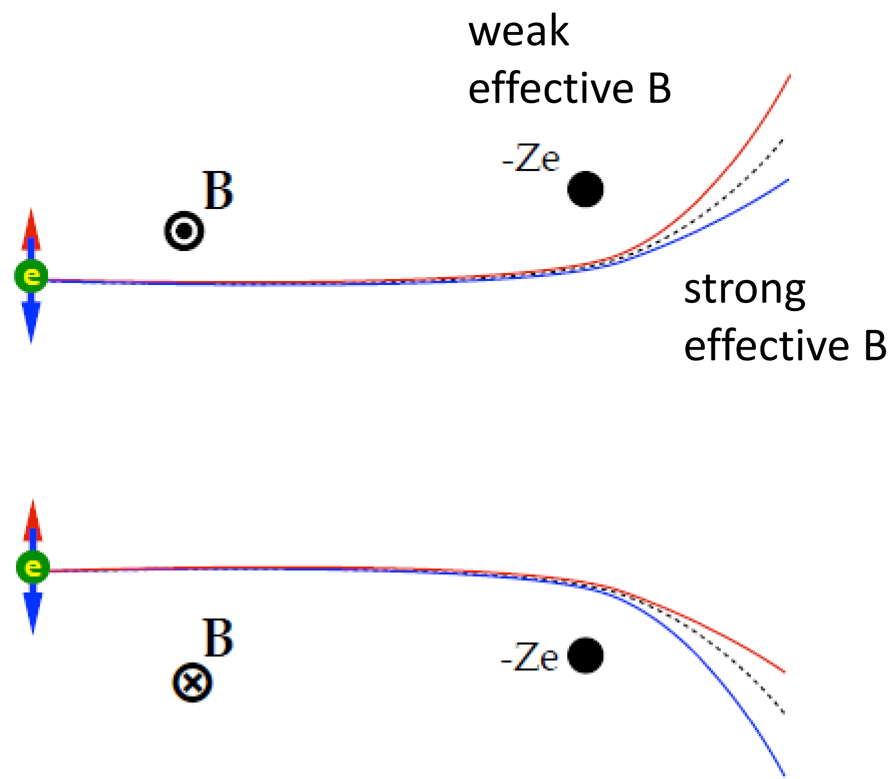


Figure from Professor Xiao Jiang's lecture notes

Explanation to SH: Spin-orbit coupling

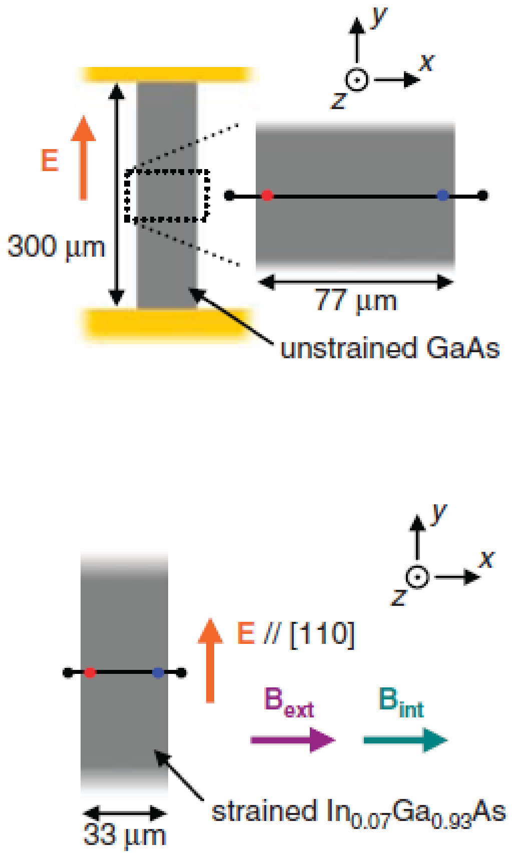
Hall effect -- Lorenz force drives different charges into different directions
spin Hall effect -- the Spin-orbital coupling effect drives different “spin charges”





Figures from Professor Xiao Jiang’s lecture notes

Experimental realization of SH



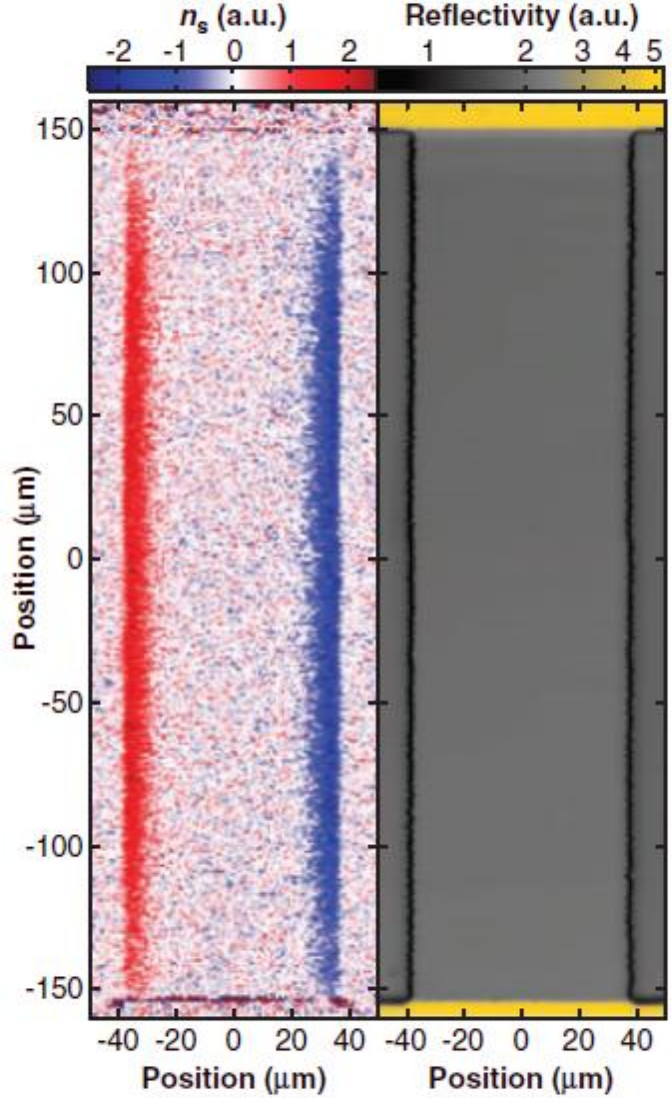
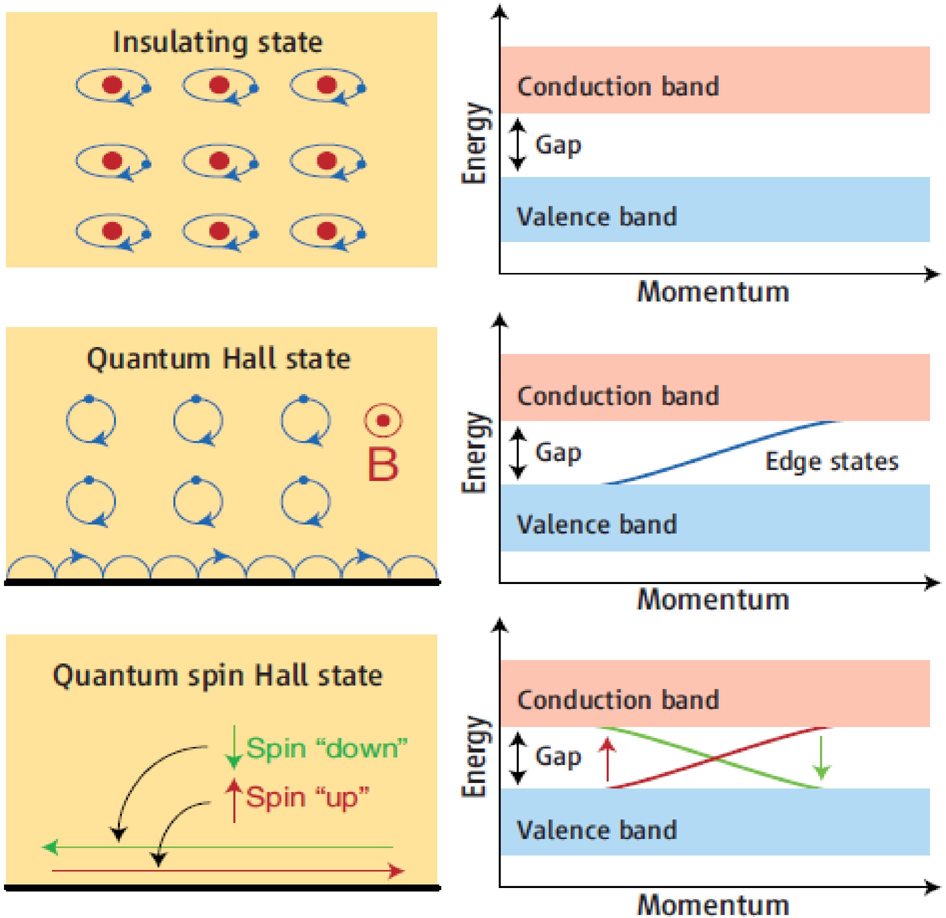
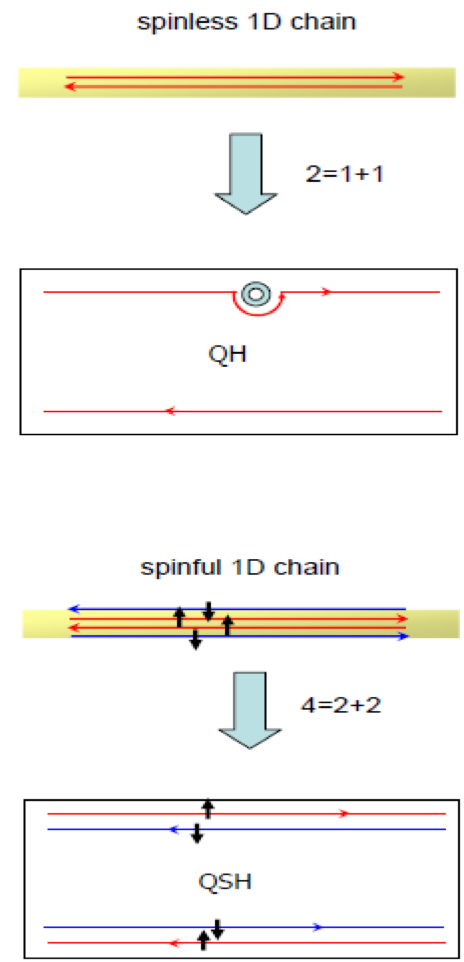


Figure from Science 306, 1910 (2004)

Quantum spin Hall effect





The number of edge channels in the sample is directly related to the value of the quantum Hall conductance

Figure from Science 301, 1348 (2003)

Figure from Physics Today Feb 2010

Experimental realization of QSH

König et al. Succeeded in observing QSH in HgTe Quantum Wells in 2007

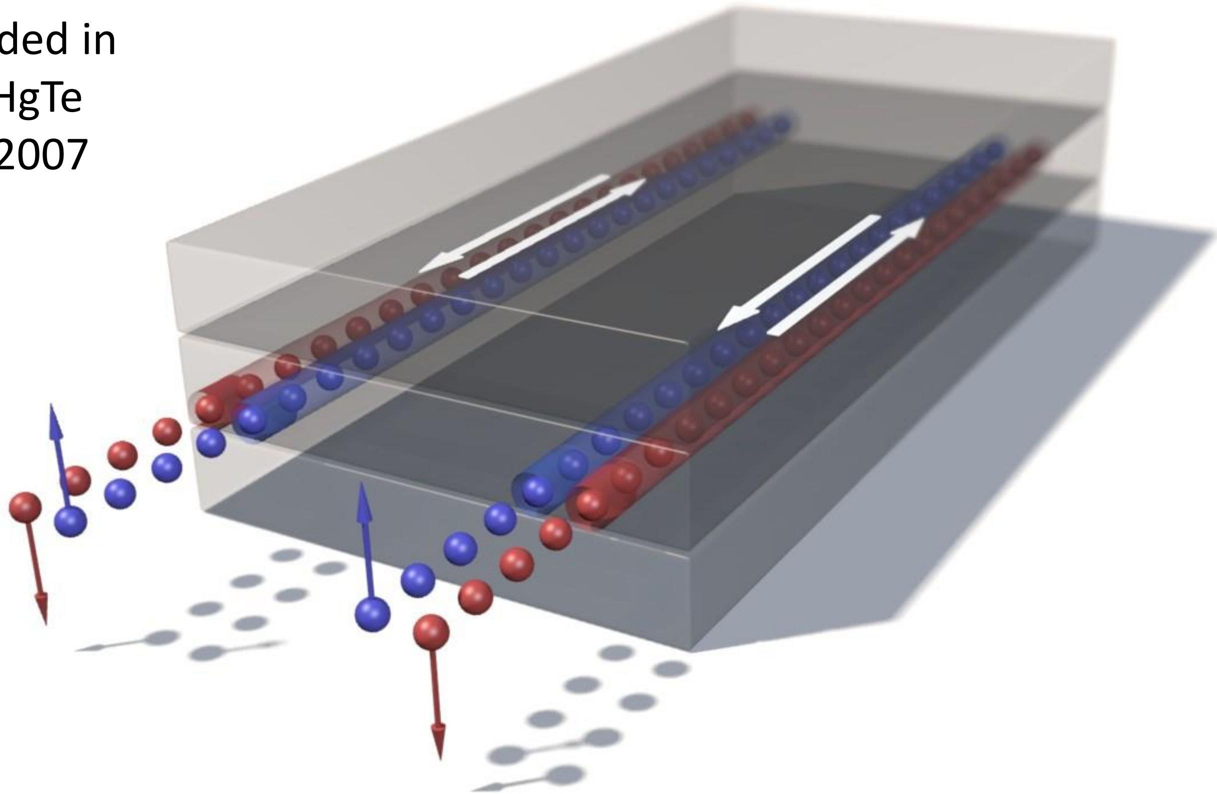


Figure from Science 318, 766 (2007)

mathematical formulation of QSH

Hamiltonian of electrons in QSH

the Hamiltonian due to the external field

Quantum Hall Effect $H_L = \boldsymbol{A} \cdot \boldsymbol{p}$

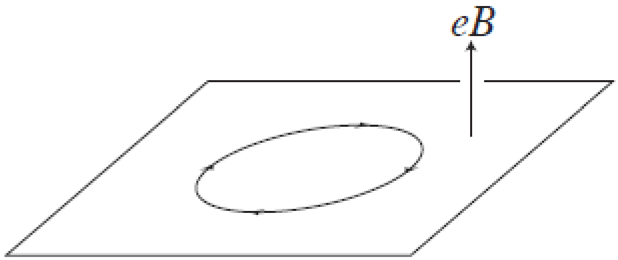
Spin Quantum Hall effect $H_{SO} = (\boldsymbol{p} \times \boldsymbol{E}) \cdot \boldsymbol{\sigma}$

Sigma is the Pauli spin matrix

Landau Level

Take the symmetric gauge (rotational invariance)

$$\vec{A} = (A_x, A_y) = \frac{B}{2}(-y, x)$$



$$H = \frac{1}{2m}\vec{\Pi}^2 = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2$$

Hamiltonian

$$[\Pi_x, \Pi_y] = [p_x - \frac{e}{c}A_x, p_y - \frac{e}{c}A_y] = i\frac{e\hbar}{c}(\nabla_x A_y - \nabla_y A_x) = i\frac{e\hbar}{c}B_z$$

$$a = \sqrt{\frac{c}{2e\hbar B}}(\Pi_x + i\Pi_y),$$
$$a^\dagger = \sqrt{\frac{c}{2e\hbar B}}(\Pi_x - i\Pi_y).$$

$\longrightarrow$

$$H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$E = \hbar\omega\left(N + \frac{1}{2}\right)$$

Taken from professor Hitoshi Murayama's lecture notes

Landau Level

N is one quantum number
the other quantum number is the center of the cyclotron motion

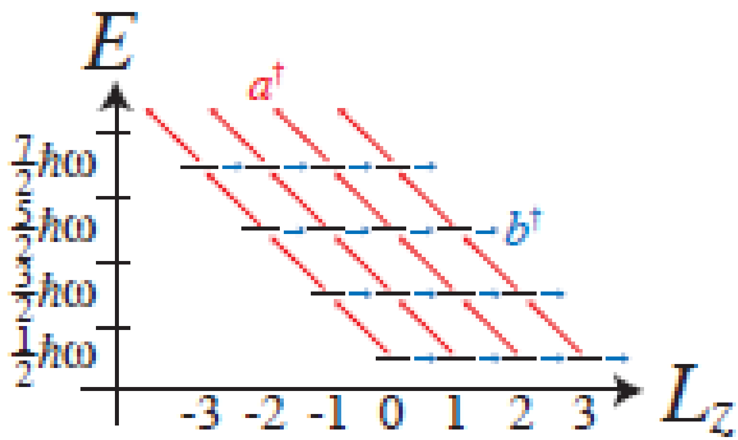
$$X = x + \frac{v_y}{\omega} = x + \frac{c}{eB}\left(p_y - \frac{e}{c}A_y\right)$$

$$b = \sqrt{\frac{eB}{2\hbar c}}(X - iY)$$

$$Y = y - \frac{v_x}{\omega} = y - \frac{c}{eB}\left(p_x - \frac{e}{c}A_x\right)$$

$$b^\dagger = \sqrt{\frac{eB}{2\hbar c}}(X + iY)$$

Two dimensional harmonic oscillator -> Two sets of ladder operators



Taken from professor Hitoshi Murayama's lecture notes

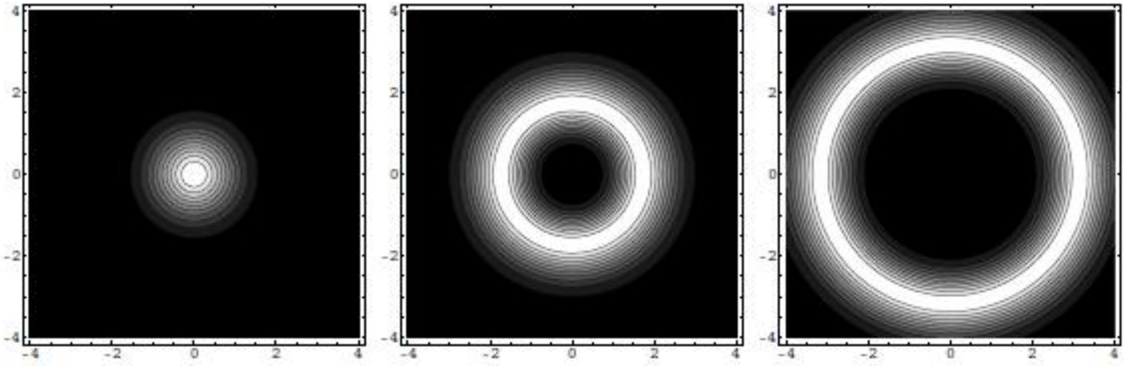
Landau Level

Ground state wave function

coordinate transformation for simplicity cause $z = x + iy,\ \bar{z} = x - iy$

$$\partial = \frac{\partial}{\partial z} = \frac{1}{2}\left(\nabla_x - i\nabla_y\right), \qquad \bar{\partial} = \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\nabla_x + i\nabla_y\right) \longrightarrow a^\dagger = -i\sqrt{\frac{\hbar c}{2eB}}\left(2\partial - \frac{eB}{2\hbar c}\bar{z}\right)$$

$$\langle z, \bar{z}|a|0\rangle = -i\sqrt{\frac{\hbar c}{2eB}}\left(2\bar{\partial} + \frac{eB}{2\hbar c}z\right)\psi(z,\bar{z}) = 0 \longrightarrow \psi(z,\bar{z}) = f(z)e^{-eB\bar{z}z/4\hbar c}$$



The ground state wave functions with n = 0, 3, and 10

Taken from professor Hitoshi Murayama's lecture notes

Spin-orbit coupling Hamiltonian

Electric field? shear strain gradients can play a similar role

$$\epsilon_{xy} \leftrightarrow E_z; \qquad \epsilon_{xz} \leftrightarrow E_y; \qquad \epsilon_{yz} \leftrightarrow E_x$$

Take a simple case as an example

$$\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = \epsilon_{xy} = 0 \quad \epsilon_{zx} = gy \qquad \epsilon_{yx} = gx$$

$$x' = \frac{1}{\sqrt{2}}(x - y), y' = -z, z' = \frac{1}{\sqrt{2}}(x + y)$$

Thus $$H = \frac{p^2}{2m} + \frac{C_3}{\hbar} g y' p_{x'} \sigma_{z'} + D y^2$$

$$H = \begin{pmatrix} H_\uparrow & 0 \\ 0 & H_\downarrow \end{pmatrix}$$ where $$H_{\downarrow,\uparrow} = \sqrt{\frac{D}{2m}}[p_x^2 + p_y^2 + x^2 + y^2 \pm R(xp_y - yp_x)]$$

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Quantum Spin Hall effect

$$a = \partial_{z^*} + \frac{z}{2}, \qquad a^\dagger = -\partial_z + \frac{z^*}{2}$$

$$b = \partial_z + \frac{z^*}{2}, \qquad b^\dagger = -\partial_{z^*} + \frac{z}{2}$$

$$H_{\downarrow,\uparrow} = 2\sqrt{\frac{D}{2m}}\left[\left(1 \mp \frac{R}{2}\right)aa^\dagger + \left(1 \pm \frac{R}{2}\right)bb^\dagger + 1\right]$$

$$L_z = bb^\dagger - aa^\dagger$$

$$\phi_n^\uparrow(z) = \frac{z^n}{\sqrt{\pi n!}}\exp\left(\frac{-zz^*}{2}\right) \qquad \phi_m^\downarrow(z) = \frac{(z^*)^m}{\sqrt{\pi m!}}\exp\left(\frac{-zz^*}{2}\right)$$

$$L_z\phi_n^\uparrow(z) = n\phi_n^\uparrow(z) \qquad L_z\phi_m^\downarrow(z) = -m\phi_m^\downarrow(z)$$

and their charge conductance is quantized in units of –e^2/h

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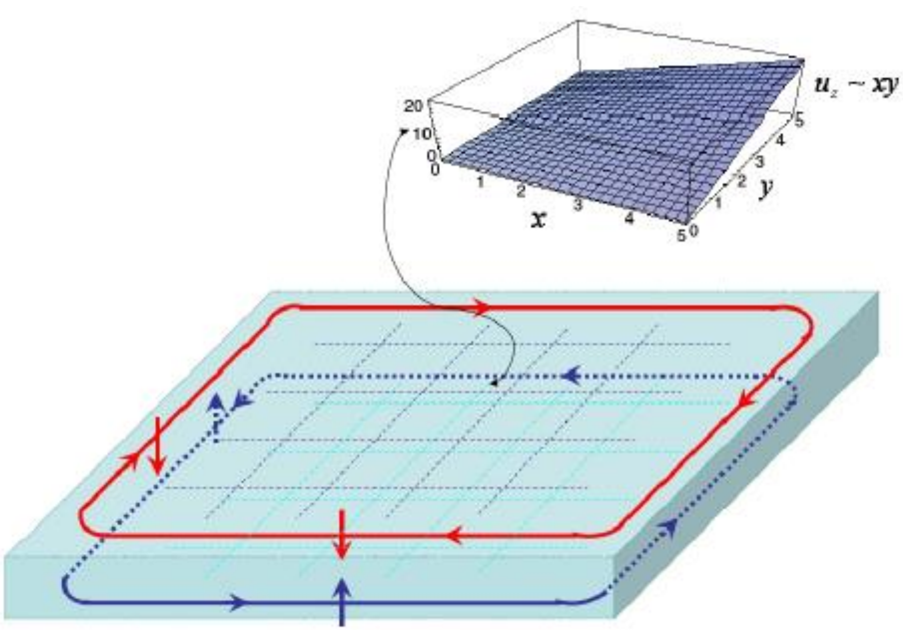
Quantized spin cunductance



Total charge conductance vanishes

spin conductance is quantized

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Scheme for Experimental Verifying

estimate the Landau level gap and the strain

R=2 $\Delta E_{Landau} = C_3 g$

g being the magnitude of the strain gradient
C_3/h is 8×10^5 m/S for GaAs by experiment

For a gap of 1 mK, we hence need a strain gradient or 1% over 60 μm.

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Thank you!