角动量 要 × m 必 必

重要概念

1) 一对力做功之和与参考系无关

$\vec F\cdot(\vec{r_1}+\vec v t)+(-\vec F)\cdot(\vec{r_2}+\vec v t)$

$=\vec F\cdot(\Delta\vec{r_1}-\Delta\vec{r_2})$

S 平动参考系 S'

$W=\int \vec{F_1}\cdot\vec{v_1}\,dt+\int\vec{F_2}\cdot\vec{v_2}\,dt$

$\vec{F_1}=-\vec{F_2}$

$W'=\int\vec{F_1}\cdot\vec{v_1'}\,dt+\int\vec{F_2}\cdot\vec{v_2'}\,dt$

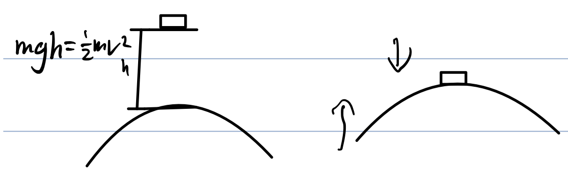
$\vec{v_1}=\vec u+\vec{v_1'}$

$\vec{v_2}=\vec u+\vec{v_2'}$

$W=\int\vec{F_1}(\vec{v_1'}+\vec u)\,dt+\int\vec{F_2}(\vec{v_2'}+\vec u)\,dt$

$=0+W'$

2) 关于势能是属于体系的



质心参考系

$$\begin{cases} mgh+0+0=0+\frac{1}{2}mv_1^2+\frac{1}{2}Mv_2^2 \\ mv_1=Mv_2 \end{cases}$$

$v_1=\sqrt{2gh}\sqrt{\frac{M}{m+M}}$

$v_2=\sqrt{2gh}$

相对速度 $v_1+v_2=\sqrt{2gh}\cdot\sqrt{\frac{M}{m+M}}\left(1+\frac{m}{M}\right)$

① 质心系中 M 动能近似为0

② …… M 加速度小，近似惯性系

③ ……中 $\Delta V=\sqrt{2gh}\sqrt{\frac{m+M}{M}}$ 与 M 系中 $\sqrt{2gh}$ 接近

4-3 功能原理 机械能守恒定律

对质点系：$A_{外}+A_{内}=\Delta E_k$

$\Rightarrow A_{外}+A_{内保}+A_{非保}=\Delta E_k$

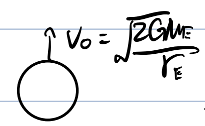
功能原理 $A_{外}+A_{非内}=\Delta E_k+\Delta E_p=\Delta E$

当 $A_{外}+A_{非内}=0$

机械能守恒 $\Delta E=0$

(1) 条件 $A_{外}+A_{非内}=0$

(2) 守恒定律是对系统而言的



$\frac{1}{2}mv^2-\frac{GMm}{r}=\frac{1}{2}mv_0^2-\frac{GMm}{r_E}$

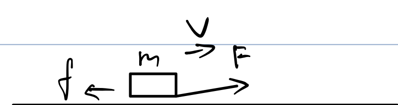
$\frac{1}{2}v^2=\frac{GM}{r}$

$v=\sqrt{\frac{2GM}{r}}=\frac{dr}{dt}$

$\int_0^t dt=\int_R^{nR}\frac{\sqrt r\,dr}{\sqrt{2GM}}$

$t=\frac{2}{3}\sqrt{\frac{R^3}{2GM}}\left(n^{\frac{3}{2}}-1\right)$

更广泛的能量守恒
—— 热力学第一定律



f做功 → 从物体m中提取能量

转成内能

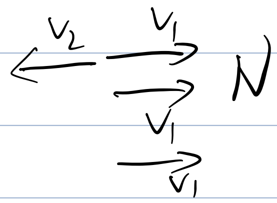
m与桌面

$A+Q=\Delta E_{total}$

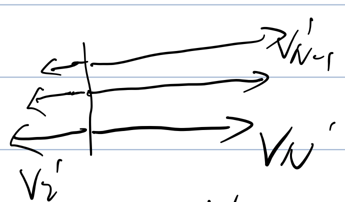
$E_{total}=E_k+E_p+E_i$

情况一：一起跳

速度



情况二：



$V_N > V_1$

$\therefore V_2' < V_2$ 不成立

4-4 柯尼西定理

质心参考系 $\sum \vec{F} = Ma_c$

与一般参考系		质心参考系 S'
势能 U	$U=U'$	U'
动能 E_k	$E_k=\frac{1}{2}MV_c^2+E_k'$	E_k'
动量 P		$P'=0$
角动量 L	$L=\vec{r_c}\times\vec{P}+\vec{L}'$	L'

推导过程

$$\vec{V_c}=\frac{\sum m_i\vec{v_i}}{\sum m_i}$$

$$E_k=\sum\frac{1}{2}m_i(\vec{V_{ic}}+\vec{V_c})^2$$

$$=\sum\frac{1}{2}m_i\vec{V_{ic}}^2+\frac{1}{2}M\vec{V_c}^2$$

$$E_c=\frac{1}{2}\sum m_i(V_c)^2$$

$$E_k'=\sum\frac{1}{2}m_i(\vec{V_i}-\vec{V_c})^2$$

$$=\frac{1}{2}\sum m_i(\vec{V_i}-\vec{V_c})^2$$

$$=\frac{1}{2}\sum m_i(V_i^2-2\vec{V_i}\cdot\vec{V_c}+\vec{V_c}^2)$$

$$=\frac{1}{2}\sum m_i\vec{V_i}^2-\frac{1}{2}\sum 2m_i\vec{V_i}\vec{V_c}+\frac{1}{2}\sum m_i\vec{V_c}^2$$

$$=\frac{1}{2}\sum m_i\vec{V_i}^2-\frac{1}{2}MV_c^2$$

第五章 角动量

$\vec F = m\vec a$

$\Rightarrow \vec F = \frac{d\vec p}{dt}$ $\left(\begin{array}{l}\vec M = \vec r\times\vec F\\ \vec L = \vec r\times\vec p\end{array}\right)$

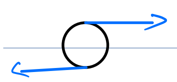
$\vec M = \frac{d\vec L}{dt}$

$\vec r\times\vec F = \underbrace{\frac{d\vec r}{dt}\times\vec p}_{=0} + \vec r\times\frac{d\vec p}{dt} = \vec r\times\frac{d\vec p}{dt}$

5.1 力矩

① 与参考点选取有关

$\vec M = \vec r\times\vec F$



$\vec F = 0$ $\vec M \neq 0$

② 去F的z分量

③ 力矩计算

z轴上任取一点为原点，计算: $\vec M = \sum \vec r_i\times\vec F_i$

④ 内力矩对总力矩贡献为零

$\vec F = m\vec a$ $\quad$ $\vec L = \vec r\times\vec p$

$\downarrow\uparrow$ $\quad$ $\vec M = \vec r\times\vec F$

$\vec F = \frac{d\vec p}{dt}$ $\quad$ $\vec M = \frac{d\vec L}{dt}$

$\vec L = \vec r\times\vec p \Rightarrow \vec M = \frac{d\vec L}{dt}$

质点系

$\vec L = \sum \vec r_n\times\vec p_n$

$\vec M = 0$ $\quad$ $\vec L$ = 恒矢量

$\vec L = \sum \vec r_i\times\vec p_i = \sum(\vec r_c + \vec r_{ic})\times m_i(\vec v_c + \vec v_{ic})$

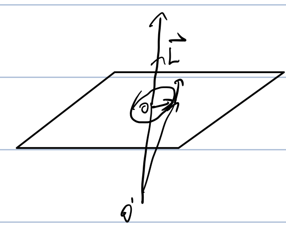
$= \sum m_i\vec r_c\times\vec v_c + \sum \vec r_{ic}\times\vec v_c m_i$

$+ \sum m_i\vec r_c\times\vec v_{ic} + \sum m_i\vec r_{ic}\times\vec v_{ic}$

$= \vec r_c\times\vec p_c + \vec L'$

$\sum m_i\vec r_c\times\vec v_c = 0$

$(\underbrace{\sum m_i\vec r_{ic}}_{0})\times\vec v_c = 0$



$\vec L = \vec r\times\vec p$

O: L 只有z分量

O': L'为倾斜，z分量恒

$\vec M = \frac{dL_z}{dt}$

$mg\cot\alpha = \frac{mv^2}{h\tan\alpha}$

$\sqrt{hg} = v$

$2v_0h = vH$

$mg(H-h) = \frac12 mv_0^2 - \frac12 mv^2$ $\quad$ $\frac{gH^2}{2(H+h)} = gh$

$g(H-h) = 2v_0^2 - \frac12\frac{4v_0^2h^2}{H^2}$ $\quad$ $H^2 = 2hH+2h^2$

$g(H-h) = 2\frac{(H-h)(H+h)}{H^2}v_0^2$ $\quad$ $x^2 = 2x+2$

$x^2-2x-2=0$

$\frac{2\pm\sqrt{4+8}}{2} = 1+\sqrt3$

一、万有引力定律

$\vec F = -G\frac{m_1m_2}{r_{12}^3}\vec r_{12} = -G\frac{m_1m_2}{r_{12}^2}\hat r_{12}$

$\vec r_{12}$ 是 m_1 指向 m_2 的矢量

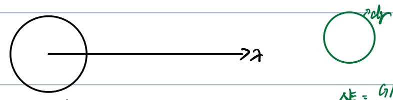
列宁：一切科学的抽象，都是更深刻、更正确、更完全地反映着自然

二、引力叠加原理

$\vec F_m = \vec F_{mm_1} + \vec F_{mm_2} + \cdots$

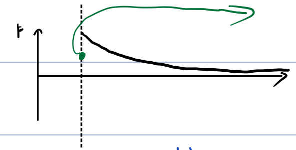
三、物体之间的引力 微积分思想

四、两个壳层定理



$\Delta E = -\frac{GMm}{2(r+dr)} + \frac{GMm}{2r} = F\,dr$

$\frac{GMm}{2r^2} = F$



六、引力势能

$$E_p = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{Gm_im_j}{r_{ij}} + \frac{1}{2}\sum_{i=1}^{N}\frac{Gm_i^2}{r_{ij}}$$

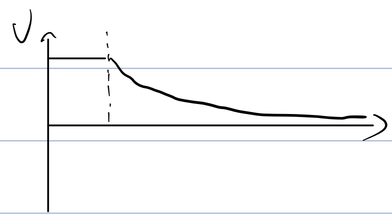
$$= -\sum_{i=1}^{N}\sum_{j=i+1}^{N}$$

N个质点在∞处（因两者相距无穷远）

$$\begin{pmatrix} \times & -\frac{Gm_1m_2}{r_{12}} & -\frac{Gm_1m_3}{r_{13}} \\ & \times & -\frac{Gm_2m_3}{r_{23}} \\ & & \times \end{pmatrix}$$

$$E_p = \sum_{\substack{i=1 \\ j\neq i}}^{N} -G\frac{m_im_j}{r_{ij}}$$

$$E_p = \sum_{\substack{i,j=1 \\ i\neq j}} -\frac{1}{2}G\frac{m_im_j}{r_{ij}}$$



二体问题与约化质量

二体：由相互作用的两个物体组成系统

- 质心运动
- 质心参考系

1) m_1运动 → m_2运动

2) m_2 → m_1运动

(3) m_2相对m_1运动

如何处理

A) 相对运动 系在m_2上

列方程

B) 直接列方程 → 相对运动

$m_1\ddot{\vec{r}}_1 = \vec{F}_1 \qquad \vec{F}_1 = -\vec{F}_2$

$m_2\ddot{\vec{r}}_2 = \vec{F}_2$

$\frac{m_1\ddot{\vec{r}}_1 + m_2\ddot{\vec{r}}_2}{m_1+m_2} = 0$

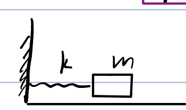
$\boxed{① \ \ddot{\vec{r}}_c = 0}$

$\ddot{\vec{r}}_1 = \frac{\vec{F}_1}{m_1} \qquad \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2 = \frac{\vec{F}_1}{m_1} - \frac{\vec{F}_2}{m_2}$

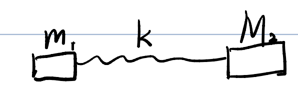
$\ddot{\vec{r}}_2 = \frac{\vec{F}_2}{m_2} \qquad \frac{m_1m_2}{m_1+m_2}\vec{a} = \vec{F}_1$

$\boxed{\mu = \frac{m_1m_2}{m_1+m_2}}$

一：



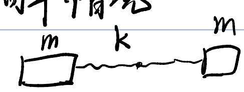
II：



$\mu = \frac{m_1m_2}{m_1+m_2}$

$\omega = \sqrt{\frac{k}{\mu}}$

III：简单情况



$\omega = \sqrt{\frac{2k}{m}}$

$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{2k}{m}}$

(1) $\frac{1}{2}\mu u^2 = \frac{1}{2}k\Delta x^2 \Rightarrow \Delta x = \sqrt{\frac{\mu}{k}}u = \sqrt{\frac{m_1m_2}{(m_1+m_2)k}}u$

(2) $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{\mu}{k}} \Rightarrow \frac{\pi}{2}\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}$

(3) $s = v_ct - \frac{\Delta x\, m_2}{m_1+m_2}$

$= \frac{m_2}{m_1+m_2}\cdot\sqrt{\frac{m_1m_2}{(m_1+m_2)k}}\cdot u\left(\frac{\pi}{2}-1\right)$

刚体

刚体：在外力作用下 形状大小都不发生变化的物体

刚体运动：平动、固定轴、平行平面运动、定点转动、一般运动

转动惯量

$\vec F = m\vec a \Rightarrow \tau_z = I\alpha$

$\vec F_\tau = m\vec a_\tau$

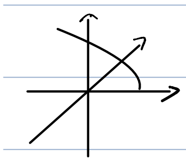
$F_\tau \cdot r = mr^2\alpha$

$I = \sum \Delta m_i r_i^2$

$= \int r^2 dm$

圆柱：$\frac{1}{2}mR^2$

均质球壳



$I_z = \sum \Delta m_i (x_i^2+y_i^2)$

$+$

$I_y = \sum \Delta m_i (y_i^2+z_i^2)$

$+$

$I_x = \sum \Delta m_i (x_i^2+z_i^2)$

$\parallel$

$2\sum \Delta m_i r_i^2$

平行轴定理：$I = I_c + md^2$

$I_o = \sum m_i \vec{r_o}^2$

$= \sum m_i (\vec d + \vec r_c)^2$

$= \sum m_i d^2 + 2\sum m_i \vec d \cdot \vec r_c + \sum m_i \vec r_c^2$

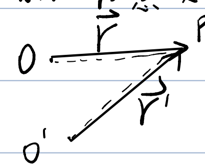
$= I_c + md^2$

薄板正交轴定理

$I_x + I_y = I_z$

瞬时轴，瞬心

1. 刚体上任意一点角速度相同



$\vec v_p = \vec v_o + \vec\omega \times \vec r$

$\vec v_p = \vec v_{o'} + \vec\omega' \times \vec r'$

$\vec v_{o'} = \vec v_o + \vec\omega \times \vec r_{oo'}$

$\vec\omega \times \vec r_{oo'} + \vec\omega' \times \vec r' = \vec\omega \times \vec r$

2. 刚体必有一瞬时轴
3. 轴在刚体或空间中
4.
5. 不同时刻有不同瞬轴
6. 瞬加速度一般不为零

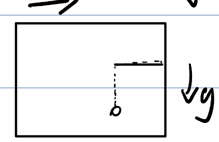
角动量守恒定律

进动 (The Spinning Top)

$\frac{d\vec L}{dt} = \vec\tau$

$\omega_p = \frac{mgr}{I\omega}$

$\vec a$ t时刻停止



$E_{k_2} + mgR(1-\cos\alpha) - maR\sin\alpha = 0+0$

$E_{k_2} = maR\sin\alpha - mgR(1-\cos\alpha)$

$E_{k_2} + mgR(1-\cos\alpha) = E_{k_3} + 2mgR$

$maR\sin\alpha = E_{k_3} + 2mgR$

$a = \frac{E_{k_3}+2mgR}{mR\sin\alpha}$

$= \frac{\frac{5}{2}g}{\sin\alpha}$

流体力学

流体及宏观物理性质

不可压缩流体

层流、湍流

流线 ⇒ 流线不相交

细流管（极限为流线）

定常 / 不定常

⇓ $\vec{v}(x,y,z)$ ⇓ $\vec{v}(x,y,z,t)$

$\rho S v = C \Rightarrow Sv = C$

理想流体 ⇒ 不可压缩，无粘滞力

$W = F_1 v_1 \Delta t - F_2 v_2 \Delta t$

$= P_1 \Delta V - P_2 \Delta V$

$\Delta E = (\frac{1}{2} m v_2^2 + mgh_2) - (\frac{1}{2} m v_1^2 + mgh_1)$

$W = \Delta E \Rightarrow \frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2 = \frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1$

对同一流管 $\frac{1}{2}\rho v^2 + \rho g h + P = C$ 伯努利方程

⇓ 理想流体 定常流动

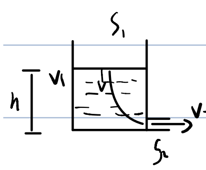
实际流体：1、粘滞流体

2、粘滞定律

$f = \eta S \frac{dv}{dy}$

3、粘性流体伯努利方程

$\frac{1}{2}\rho v_2^2 + \rho g h_2 + P_2 = \frac{1}{2}\rho v_1^2 + \rho g h_1 + P_1 + w$



① 1、慢流
2、理想流体

(Ⅱ) 流线 流动过程

(Ⅲ) 列方程 $P_1 = P_0$
$P_2 = P_0$

$S_1 v_1 = S_2 v_2$

$P_0 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_0 + \frac{1}{2}\rho v_2^2$

$v_1 = S_2\sqrt{\frac{2gh}{S_1^2 - S_2^2}}$

$v_2 = S_1\sqrt{\frac{2gh}{S_1^2 - S_2^2}}$

$-v_1 = \frac{dh}{dt} = -S_2\sqrt{\frac{2gh}{S_1^2 - S_2^2}}$

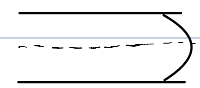
$-2\sqrt{h_0} = -\frac{S_2\sqrt{2g}}{\sqrt{S_1^2 - S_2^2}} t$

$h = \frac{S_2^2 g}{2(S_1^2 - S_2^2)} t^2$

$t = \sqrt{\frac{2(S_1^2 - S_2^2)h}{S_2^2 g}}$

4、泊肃叶公式

$Q = \frac{\pi (P_1 - P_2) R^4}{8\eta l}$



$P_1 \pi r^2 - P_2 \pi r^2 = -\eta L 2\pi r \cdot \frac{dv}{dr}$

$\frac{(P_1 - P_2) r}{2\eta l} dr = -dv$

$\int_v^0 dv = \int_r^R \frac{(P_1 - P_2) r}{2\eta l} dr$

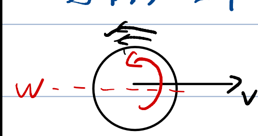
$v = \frac{(P_1 - P_2)}{4\eta l}(R^2 - r^2)$

$Q = \int_0^R 2\pi r\, dr \frac{(P_1 - P_2)}{4\eta l}(R^2 - r^2)$

$= \frac{\pi (P_1 - P_2)}{8\eta l} R^4 = \frac{\Delta P}{R_f}$

$R_f = \frac{8\eta L}{\pi R^4}$

马格努斯效应



康达效应

5、流体中运动

粘滞阻力 1) $f = kv$

2) $f = 6\pi\eta R v \Rightarrow v_T =$

压差阻力

6、层流、湍流、雷诺数

2000
3000
$R_e = \frac{\rho v r}{\eta}$

雷诺相似准则 $v\nearrow$ $r\downarrow$ 流场变

振动

- 振动
- 机械振动
- 简谐振动
- 运动学描述
- $\vec{F} = m\vec{a} \longrightarrow \ddot{x} + \omega x = 0$
- 阻尼振动、受迫振动
- 概念推移 $A\cos(\omega t + \varphi)$

(I) ω 圆频率，固有频率

(II) A, φ 由初始(边界)条件确定

$$A = \sqrt{x^2(0) + \frac{\dot{x}^2(0)}{\omega^2}}$$

$$tg\varphi = \frac{-\dot{x}(0)/\omega}{x(0)}$$

(III) A 振幅

$\omega t + \varphi$ 相位

φ 初相位

(IV) ω $\quad f = \frac{\omega}{2\pi}$

$\downarrow$ $\quad \frac{1}{f} = T$

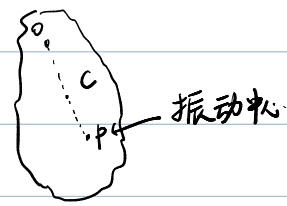
与振动无关

$\downarrow$

固有频率

复摆

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$



$\omega_f = \sqrt{\omega_0^2 - \delta^2}$

阻尼振动：$x = A_0 e^{-\delta t}\cos(\omega_f t + \varphi_f)$

受迫振动：$x = A_0 e^{-\delta t}\cos(\omega_f t + \varphi_f) + B\cos(\omega t - \varphi)$

品质系数 $Q_1 = 2\pi\frac{E}{\Delta E} \approx \frac{\omega_0}{2\delta}$

$Q_2 = \frac{\omega_r}{2\Delta\omega} = \frac{\omega_0}{2\delta}$

$B = \frac{f}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$ $\quad tg x = \frac{2\delta\omega}{\omega_0^2 - \omega^2}$

$B_{max} = \frac{f}{2\delta\sqrt{\omega_0^2 - 2\delta^2}}$ $\quad \omega_r = \sqrt{\omega_0^2 - 2\delta^2}$

速度共振 $\omega_r = \omega_0$

拍频 $\nu = \nu_1 - \nu_2$

波动：扰动与振动的传播

能量传播的形式

$$\nabla^2 f - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0$$

nabla

机械波：振动在介质中传播

质元与质元相互作用

使振动得以传播

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

解只能是 $f(x+vt)$ $f(x-vt)$ 与其线性组合

$y = f(x - vt)$

$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial w}\frac{\partial w}{\partial x} = \frac{\partial y}{\partial w}\cdot 1$

$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) = \frac{\partial^2 y}{\partial w^2}\cdot 1$

$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial w}(-v)$

$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial w^2}(-v)^2$

$T\frac{\partial^2 y}{\partial x^2} = \rho\frac{\partial^2 y}{\partial t^2}$

$\frac{\partial^2 y}{\partial x^2} = \frac{\rho}{T}\frac{\partial^2 y}{\partial t^2}$

$v = \sqrt{\frac{T}{\rho}}$

一维弦单位时间传播能量平均值

$dk = \frac{1}{2}dm\,u_y^2 = \frac{1}{2}\mu dx\,u_y^2$ $\quad y = y_m\cos(kx-\omega t)$

$= \frac{1}{2}\mu dx\,y_m^2\omega^2\cos^2(kx-\omega t)$

$dV \approx kx\delta x = T\Delta L = T(dl - dx)$

$dl = \sqrt{dx^2+dy^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$= dx\left(1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2\right)$

$\Rightarrow dU = \frac{1}{2}\left(\frac{dy}{dx}\right)^2 dx\cdot T$

$= \frac{1}{2}\underbrace{T}_{v^2\mu}dx\,y_m^2k^2\cos^2(kx-\omega t)$

$v = \frac{\omega}{k}$ $\quad v = \sqrt{\frac{T}{\mu}}$

$= \frac{1}{2}v^2\mu dx\,y_m^2k^2\cos^2(kx-\omega t)$

$= \frac{1}{2}\mu dx\,y_m^2\omega^2\cos^2(kx-\omega t)$

$\frac{dE}{dt} = \mu v y_m^2\omega^2\left[\overline{\cos^2(kx-\omega t)}\right]$

$= \frac{1}{2}\mu v y_m^2\omega^2$

$I = \frac{P}{S} = \bar{w}u$ $\quad \bar{w} = \frac{1}{2}\rho A^2\omega^2$

$w = \rho A^2\sin^2\omega(t-\frac{x}{u})$

$I = \frac{1}{2}\rho A^2\omega^2 u$

$v = \frac{F}{\rho_l u} = \frac{F}{Z}$

波阻 $Z = \rho_l u$ (μ)

三维空间 $\Delta E = \frac{1}{2}\rho\Delta V y_m^2\omega^2$

$\frac{\Delta E}{\Delta V} = \frac{1}{2}\rho y_m^2\omega^2$

ΔS面积 $\frac{\Delta E}{\Delta V}\Delta S u = \frac{1}{2}\rho y_m^2\omega^2\Delta S v$

$I = \frac{1}{2}\rho y_m^2\omega^2 v$ $\quad \rho_l = \frac{\Delta m}{\Delta l} = \rho\Delta S$

一、波叠加原理

二、惠更斯原理

菲涅耳

基尔霍夫　衍射积分

三、波的干涉

(1) 振动方向相同

(2) 频率相同

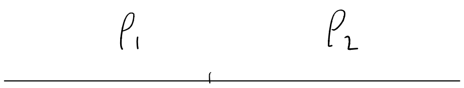
(3) 相位差恒定

四、驻波

驻波不传递能量，但能量在内部传递

五、界面反射

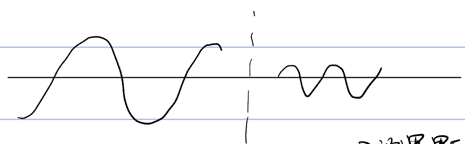
波疏→波密（半波损失）



$P_{av} = \frac{1}{2}\rho_{l_1} v_1 A^2 \omega^2$ $\quad \sqrt{T\rho_1}$ $\quad \rho_{l_2} > \rho_{l_1}$

$P_{bv} = \frac{1}{2}\rho_{l_2} v_2 A^2 \omega^2$ $\quad \sqrt{T\rho_2}$

保持能量传递守恒 ⇒ 边界问题

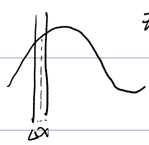


⇒ 波遇界面必有反射

$y_i = A\cos(\omega t - k_1 x)$ $\quad y_t = B_2\cos(\omega t - k_2 x + \varphi_t)$

$y_r = B_1\cos(\omega t + k_2 x + \varphi_r)$

几何边界：界面左侧 $y_1 = y_r + y_i$ $\quad y_2 = y_t$ ⇒ $y_1|_{x=0} = y_2|_{x=0}$



动力学边界：受力 y 方向 $-T\left(\frac{\partial y_1}{\partial x}\right)_{x=-\frac{\Delta x}{2}}$

$+T\left(\frac{\partial y_t}{\partial x}\right)_{x=\frac{\Delta x}{2}} = \Delta m\, a_y$

$\Delta x \to 0$ ⇒ $\left.\frac{\partial y_1}{\partial x}\right|_{x=0} = \left.\frac{\partial y_t}{\partial x}\right|_{x=0}$

$$\Rightarrow \begin{cases} A\cos\omega t + B_1\cos(\omega t + \varphi_r) = B_2\cos(\omega t + \varphi_t) \\ k_1 A\sin\omega t - k_1 B_1\sin(\omega t + \varphi_r) = k_2 B_2 \sin(\omega t + \varphi_t) \end{cases}$$

解方程：$\omega t = \frac{\pi}{2}, 2\pi + \frac{\pi}{2}$ ⇒ $-B_1\sin\varphi_r = -B_2\sin\varphi_t$

$\omega t = 0, 2\pi$ ⇒ $-B_1 k_1 \sin\varphi = B_2 k_2 \sin\varphi_t$

方程有解需 $\sin\varphi_r = \sin\varphi_t = 0$

$\varphi_r, \varphi_t = 0, \pi, 3\pi \ldots$

① $\begin{cases} \varphi_r = 0 \\ \varphi_t = 0 \end{cases}$ ⇒ $\begin{cases} A\cos\omega t + B_1\cos\omega t = B_2\cos\omega t \\ A k_1\sin\omega t - B_1 k_1 \sin\omega t = B_2 k_1 \sin\omega t \end{cases}$

⇒ $\begin{cases} A + B_1 = B_2 \\ Ak_1 - B_1 k_1 = B_2 k_1 \end{cases}$ ⇒ $\begin{cases} \frac{B_1}{A} = \frac{k_1 - k_2}{k_1 + k_2} \\ \frac{B_2}{A} = \frac{2k_1}{k_1 + k_2} \end{cases}$

Ⅰ $t = \frac{B_2}{A} = \frac{2k_1}{k_1 + k_2} = \frac{2\cdot\frac{1}{v_1}}{\frac{1}{v_1} + \frac{1}{v_2}}$ $\quad k_1 = \frac{\omega}{v_1}$ $\quad v_1 = \sqrt{\frac{F_T}{\rho_{l_1}}}$ $\quad \frac{1}{v_1} = \sqrt{\frac{\rho_{l_1}}{T}}$

$= \frac{2\sqrt{\frac{\rho_{l_1}}{T}}}{\sqrt{\frac{\rho_{l_1}}{T}} + \sqrt{\frac{\rho_{l_2}}{T}}} = \boxed{\frac{2Z_1}{Z_1 + Z_2}}$ $\quad Z_1 = \rho_{l_1} v_1 = \sqrt{T\rho_{l_1}}$

$$\boxed{r = \frac{B_1}{A} = \frac{Z_1 - Z_2}{Z_1 + Z_2}}$$

能量 $R = r^2$ $\quad T = 1 - R$

Ⅱ 如果 $Z_1 > Z_2$ $\quad r > 0$ $\quad$ 无相位跳变 无半波损失

$Z_1 < Z_2$ $\quad r < 0$ $\quad$ 半波损失

$Z_1 = Z_2$ $\quad$ 无反射

Ⅲ 固定端 $\rho_{l_2} \to \infty$ $\quad Z_2 \to \infty$ $\quad$ 波节

$r = -1$

自由端 $\rho_{l_2} \to 0$ $\quad Z_2 \to 0$ $\quad$ 波腹

$r = 1$

驻波再认识：简正模式 - 驻波 ⇒ 弦的固有频率

$f_n = n\left(\frac{v}{2L}\right)$ $\quad$ 波速 $v = \sqrt{\frac{F_T}{\rho_l}}$ ，长度 L

粗 长 重 大 ⇒ 小

细 短 轻 小 ⇒ 大

多普勒效应

观测者动 $\nu' = \frac{v_0 \pm u}{u}\nu$

波源动 $\nu' = \frac{u}{u \mp v_s}\nu$

$$\nu' = \frac{u \pm v_0}{u \mp v_s}\nu$$

波源速度运行波速不在同一直线时取投影

⇓ 非远小需考虑沿延迟